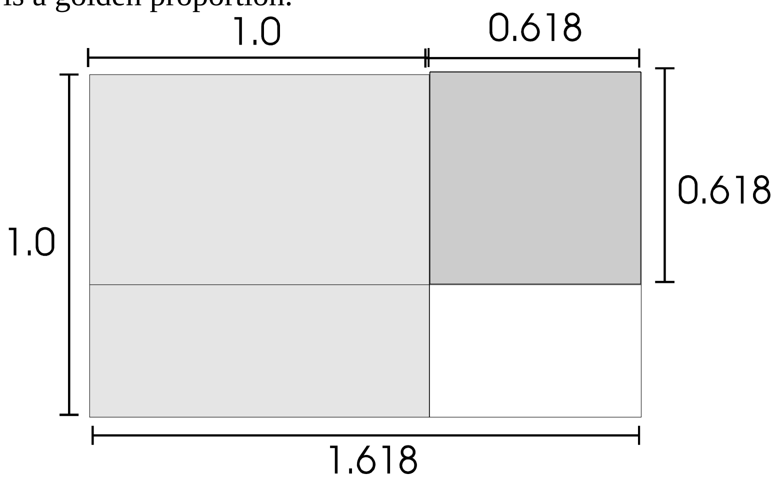
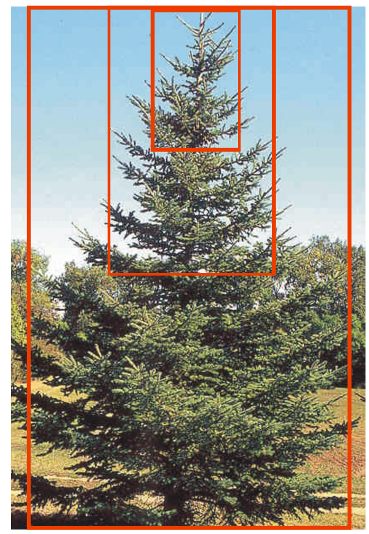
Proportions in Greek and Renaissance Art

Golden Rectangles

A Golden Rectangle is a rectangle in which the longer side is 1.618 times the shorter side, and the shorter side is 0.618 times the longer side. Many shapes in nature fill a golden rectangle. A spruce tree has golden proportions in height and width. The dragonfly's wingspan length to his body length is a golden proportion.





Spruce tree

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Golden Proportions

A proportion is the relation of one part to another. In a golden proportion, one length is 0.618 times the other length. The exact formula is:

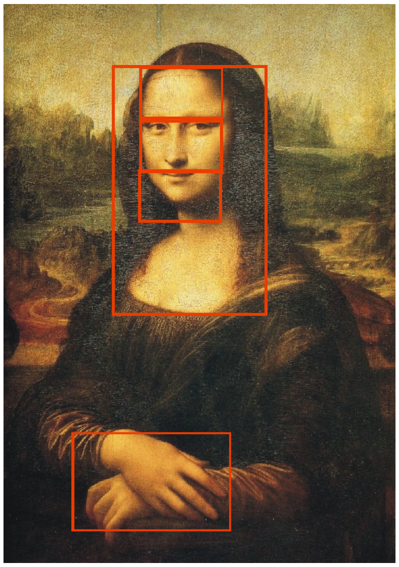
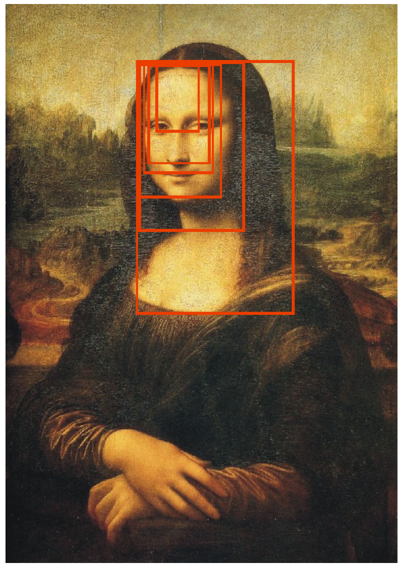
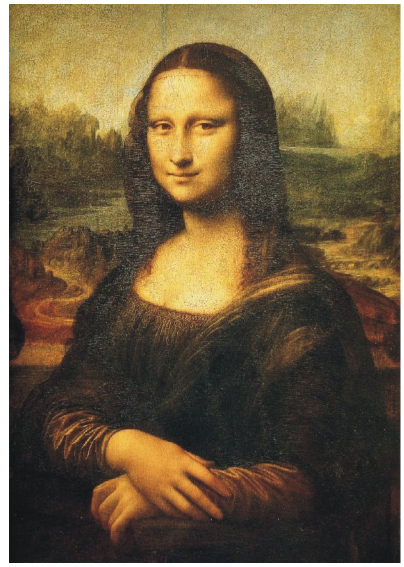
$$\frac{2}{1+\sqrt{5}}.$$

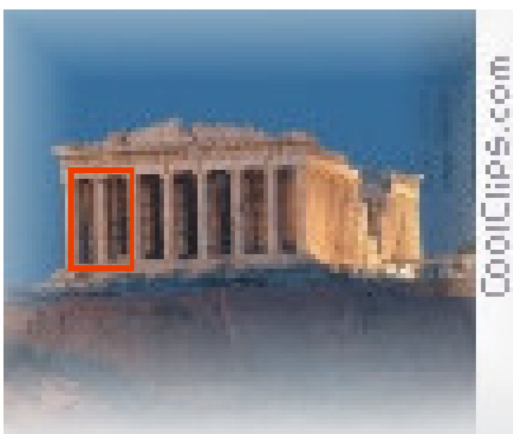
It can also be found from the Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610,...). Dividing Fibonacci numbers (number gets closer and closer to the golden proportion):

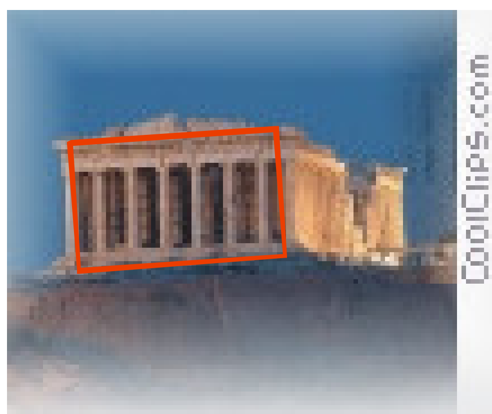
$1 \div 1$	= 1
$1 \div 2$	= 0.5
$2 \div 3$	= 0.667
$3 \div 5$	= 0.6
$5 \div 8$	= 0.625
$8 \div 13$	= 0.615
$13 \div 21$	= 0.619
$21 \div 34$	= 0.618

Golden Proportions In Greek and Renaissance Art

Since the Ancient Greek times, artists have regarded the golden proportion as one of ideal beauty. It can be found throughout in paintings (like the "Mona Lisa" http://avline.abacusline.co.uk/pictures/jpeg/pics/mona.jpg, sculptures, and architecture (like the "Parthenon"). An easy way to quickly measure golden ratios is to use the Fibonacci numbers in some measuring unit like centimeters or inches.





The following is quoted and paraphrased from
http://www.goldenmuseum.com/0305GreekArt_engl.html

As the main requirements of beauty Aristotle puts forward an order, proportionality and limitation in the sizes.

- In music Aristotle recognizes the octave as the most beautiful consonance taking into consideration that a number of oscillations between the fundamental tone and the octave is expressed by the first numbers of a natural series: 1:2.
- In poetr,y the rhythmic relations of a verse are based on small numerical ratio.
- Aristotle and Plato thought supreme beauty arose from proportions based on the "golden section".

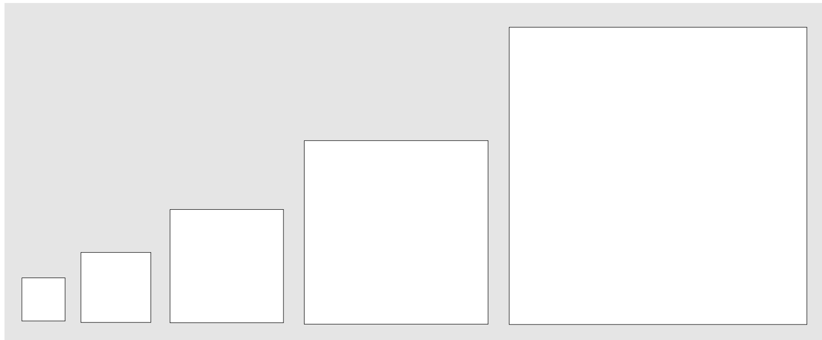
Consequences in Greek Architecture – building constructed on the basis of the golden section:

- The antique Parthenon
- "Canon" by Policlet, and Afrodita by Praksitle
- The perfect Greek theatre in Epidavre and
- the most ancient theatre of Dionis in
- The theatre in Epidavre is constructed by Poliklet to the 40th Olympiad. It was counted on 15 thousand persons. Theatron (the place for the spectators) was divided into two tiers: the first one had 34 rows of places, the second one 21 (Fibonacci numbers)! The angle between theatron and scene divides a circumference of the basis of an amphitheater in ratio: 137°,5 : 222°,5 = 0.618 (the golden proportion). This ratio is realized practically in all ancient theatres.
- Theatre of Dionis in Athens has three tiers. The first tier has 13 sectors, the second one 21 sectors (Fibonacci numbers)!. The ratio of angles dividing a circumference of the basis into two parts is the same, the golden proportion.
- From the Fibonacci series: 5, 8, 13 are values of differences between radiuses of circumferences lying in the basis of the schedule of construction of the majority of the Greek theatres. The Fibonacci series served as the scale, in which each number corresponds to integer units of Greek's foot, but at the same time these values are connected among themselves by unified mathematical regularity.

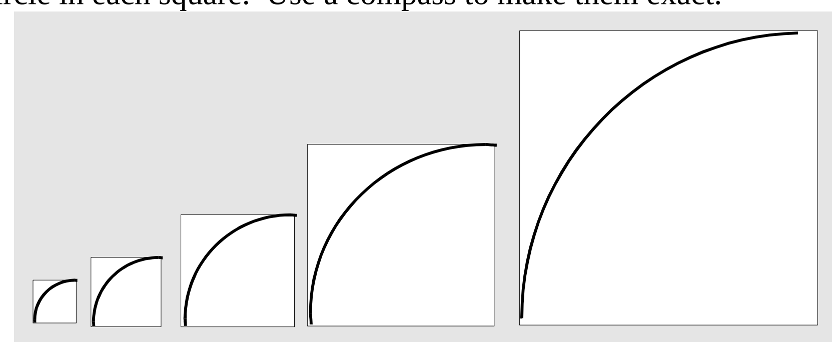
At construction of temples a man is considered as a "measure of all things: in temple he should enter with a "proud raised head ". His growth was divided into 6 units (Greek foots), which were sidetracked on the ruler, and on it the scale was put, the latter was connected hardly with sequence of the first six Fibonacci numbers: 1, 2, 3, 5, 8, 13 (their sum is equal to $32=2^5$). By adding or subtracting of these standard line segments necessary proportions of building reached. A six-fold increase of all sizes, laying aside of the ruler, saved a harmonic proportion. Pursuant to this scale also temples, theatres or stadiums are built.

Golden Spirals

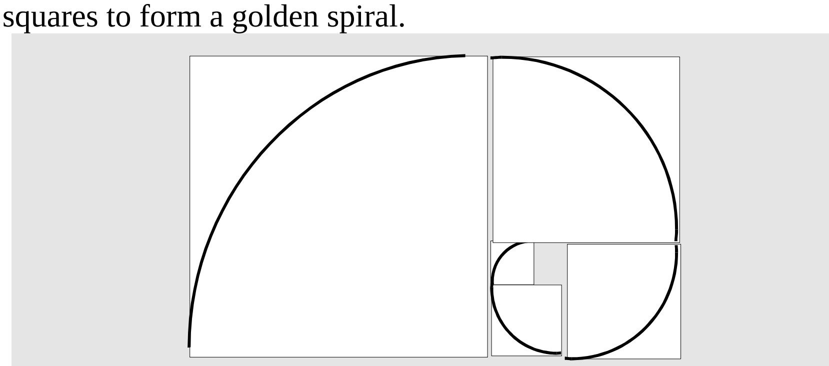
Make a small square. Then make a set of squares in which the length of the next size square is 1.618 times the length of the last square. The easiest way is to use the Fibonacci numbers -- make one 1 cm on a side, then 2, 3, 5, 8, up to 55 cm on a side.



Draw a quarter circle in each square. Use a compass to make them exact.



Then arrange the squares to form a golden spiral.



There are many examples of golden spirals in nature -- shells, horns of mountain sheep, ferns, pine cones, pussy willows, elephant tusks, some spider webs, sea horse tails, hurricanes, and galaxies to name a few.

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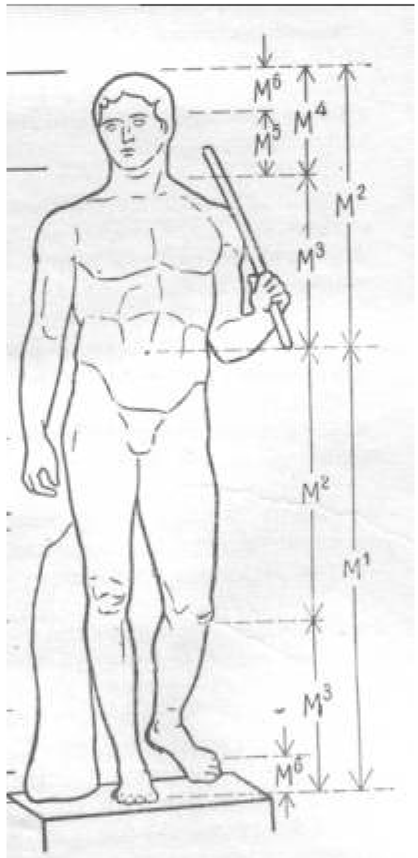

“Harmonic analysis of the Dorifor statue given in the book "Proportionality in the architecture" (1933) by the Russian architect G.D. Grimm indicates the following connections of the famous statue with the golden section $M = \tau$:

1. The first section of the Dorifor figure or its overall height $M^0 = 1$ in the proportion of the golden section $M^1 = \tau^{-1}$ and $M^2 = \tau^{-2}$ passes through a navel.
2. The second section of bottom part of a trunk $M^1 = \tau^{-1}$ and $M^2 = \tau^{-2}$ passes through the line of his knee.
3. The third section $M^3 = \tau^{-3}$ and $M^4 = \tau^{-4}$ passes through the line of his neck.”

Some Math:
M4= head height = (1-0.618)M2
M2 = upper body height = (1-0.618) body height
Head height = $(1-0.618)^2$ body height ~ 0.146 body height
Compare to Egyptian: Head Height = 1/7 body height ~ 0.143 body height