About Probability...

Pascal's Triangle

The French mathematician Blaise Pascal is credited with popularizing Pascal's triangle over 300 years ago. It is actually several thousand years old.

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              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
```

Can you figure out the pattern to fill in the next line? Try on your own before reading further.

The pattern is to start each row with a "1", then add the first two numbers from the previous row and record that sum next to the "1". Then continue adding pairs of numbers from the previous row. When all the pairs are added place another "1" at the end of the row.

Each row is symmetric or a palindrome (something that reads forwards the same as backwards). Each row has one more previous entry than the last row.

Do you see a pattern in the series of numbers that comes from totaling all the values in each row. Row 1 is 1. Row 2 is 2. Row 3 is 8. The totals follow the pattern:

2, 4, 8, 16, 32, 64, 128, 256, …

How can that pattern be represented mathematically?

Answers...

The next rows in Pascal's triangle has the following values:

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1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
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The totals follow the pattern:

2, 4, 8, 16, 32, 64, 128, 256, …

The next value in the series is always 2 times the previous value. One way to write this is:

$$x_n = 2 * x_{n-1}$$

The value at the nth place in the series is x_n. The value of the n-1 place in the series is written x_{n-1}. But that notation means that you always have to solve for all the values in the series to get a particular number. What if someone asked you "what is the 25th value in this series?" Another ways to write this is:

$$x_n = 2^{n}$$

where the exponent n means to multiply 2 by 2 n times. The 25th value in the series is $2^{25} = 33554432$.

1 chances in 32 of 3 heads and 2 tails.

2 chance in 256 that all are heads.

3 For 7 candies, there are 8 possible outcomes (all heads, 7 heads/1 tail, 6 heads/2 tails, 5 heads/3 tails, 4 heads/4 tails, 3 heads/5 tails, 2 heads/6 tails, 1 head/7 tails, all tails) and this corresponds to row 8 of the triangle.

None, if you only have 3 candies you can only have 3 possible outcomes: 3 heads, 2 heads/1 tails, and 1 tails/2 heads or

Lunchbox Math Bytes

easy to digest mathematics for your lunchbox

Probability and Statistics

Part 1: Flipping Coins

You will need to pack:

1 Pencil

A bag of M&M's™ or other candies that are flat and have distinguishable sides and won't break when flipped.

Probability

Probability

Imagine that each number on Pascal's triangle was a peg and that you could drop small balls down on the top "1" peg and as each ball hit a peg it would fall to the left half the time and fall to the right half the time. Then at each row the numbers represent the probability or chances of balls reaching that peg.

Adding the numbers in that row shows how many possible paths there are for a ball dropped on the first peg to reach that row. For example the third row has three pegs and 4 possible ways of reaching one of the pegs. The first one represents the fact that there is 1 possibility that a ball will fall to the left 2 times (once on row 1 and on row 2). There are 2 possibilities that a ball will fall to the left once and to the right once. There is 1 possibility that a ball will fall to the right 2 times. There are a total of 1+2+1 = 4 possible paths to row 3. Likewise there are a total of 1+5+10+10+5+1=32 possible paths for row 6.

Along each row the number of possible paths to a peg (the value on the triangle) divided by the total possible paths to the row (the sum of values along a row) is the probability that a ball will reach that row.

Candy Tosses

The same mathematical rules work even for coin tosses or flipping anything that is flat and has two distinguishable sides. For M&M tosses, only use ones that say "M" for "heads" and a blank for "tails."

The same probabilities are valid for coin tosses as for the ball drop. When a ball hits a peg, half the time it will fall to the right and half the time it will fall to the left. When a coin is tossed so that it lands flat, half the time it will be "heads up" and half the time it will be "tails."

The probabilities for tossing two candies are in the 3rd row of the triangle. There is 1 possibility that all are heads, 2 possibilities that 2 are heads and 2 will be tails, and 1 possibility that all are tails.

Try tossing 6 candies. The 7th row has the probabilities for the 6 possible outcomes. There is 1 chance in 64 that all 6 will be heads. 6 chances in 64 of 5 heads and 1 tails. 15 chances in 64 of 4 heads and 2 tails. 20 chances in 64 of 3 heads and 3 tails. 15 chances in 64 of 2 heads and 4 tails. 6 chances in 64 of 1 heads and 5 tails. 1 chance in 64 of all 6 tails.

For any given number of candies tossed the number of possible outcomes corresponds to the row in the triangle that has that number of entries. So, for 15 candies, look at the 16th row.

Test your knowledge

1. If you throw 5 candies what are the chances of 3 heads and 2 tails?
2. If you throw 8 candies what are the chances that all are heads?
3. If you throw 7 candies, how many different outcomes are there and which row is that on the triangle?
4. If you throw 3 candies what are the chances of 2 heads and 2 tails.

Work Space

Record the number of heads and number of tails when you flip 6 candies. Write the probability of each toss:

Heads	Tails	Probability

How do the results compare to the expected values given in Pascal's triangle?

Record the number of heads and number of tails when you flip _____ candies.

Heads	Tails	Probability