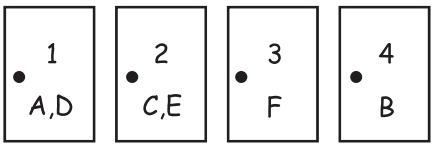
Warm-Up 17

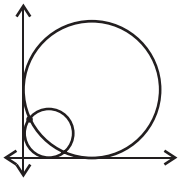
1. _____ ways

Five girls (Alexandra, Betsy, Catherine, Deyola and Emily) travel with one boy (Frank) to a math contest. They have four hotel rooms, numbered 1 through 4. Each room can hold up to two people, and the boy has to have a room to himself. How many different ways are there to assign the students to the rooms, including the way shown here?



2. _____ sq units

Two different circles are each tangent to both the x-axis and the y-axis, as shown in the figure. One of the points of intersection of the circles is (1, 5). What is the product of the lengths of their radii?



3. (_____, _____)

What point on the line $y = 2x$ is closest to the point (0, 5)?

4. _____

What is the area of regular octagon ABCDEFGH divided by the area of quadrilateral ACEG? Express your answer in simplest radical form.

5. _____

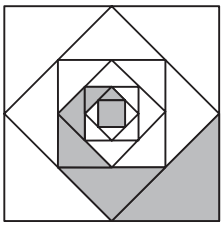
A fair coin is tossed four times, and at least one of the tosses results in heads. What is the probability that exactly two tosses result in heads? Express your answer as a common fraction.

6. _____

Many numbers can be made by adding two or more consecutive terms of the arithmetic sequence 2, 5, 8, 11, … . Two such examples are 7 = 2 + 5 and 24 = 5 + 8 + 11. What is the smallest number that can be made in at least two different ways by adding consecutive terms of this sequence?

7. _____ sq units

Each of the six smaller squares in the sequence shown is formed by joining the midpoints of the sides of the next larger square. The center square has an area of 1 square unit. What is the total area of the shaded regions? Express your answer as a mixed number.



8. _____ mph

The cities of Smallville and Largeville are 300 miles apart. Jim left from Smallville to go to Largeville at 10 a.m. Mickey left Largeville to go to Smallville at 10:30 a.m. on the same day. Jim traveled at a constant speed that was twice Mickey's constant speed, and they both arrived at a point 90 miles from Largeville at the same time. What was Mickey's constant speed, in miles per hour?

9. _____ grids

On a 5 by 5 grid of unit squares, one unit square is colored blue, one unit square is colored red, and the rest of the unit squares are white. Grids are considered different if no rotation could turn one into the other. How many different grids are there?

10. _____

Hypotenuse BC in isosceles right triangle ABC is x units, where x is a value that is a perfect square and a perfect cube greater than 1. If AB can be expressed as $\frac{32k\sqrt{5}}{5}$ units, what is the least possible value of k? Express your answer in simplest radical form.