Warm-Up 18

1. _____ Rectangle ABCD has points E and F on sides AB and CD, respectively. If AE = $\frac{1}{3}$ AB and CF = $\frac{1}{4}$ CD and segments DE and BF intersect diagonal AC at G and H, respectively, what is the ratio AG:GH:HC? Express your answer in the form $a:b:c$, where a, b and c are relatively prime positive integers.

2. _____ units A circle with radius 1 unit lies in the first quadrant and is tangent to both the x- and y-axes. A second larger circle lies in the first quadrant, is tangent to both axes and is externally tangent to the first circle. What is the radius of the second circle? Express your answer in simplest radical form.

3. _____ terms How many terms are in the expansion of the expression $[(3x + 2y)^2 (3x - 2y)^2]^3$ after it is simplified to lowest terms?

4. _____ When 97, 151 and 241 are each divided by a positive integer K, the remainder is the same. What is the largest possible value of K?

5. _____ units Equilateral triangle ABC has a side length of 6 units. Point D lies on segment BC such that DC = 2(BD). What is the length of the altitude of triangle ADC from point C? Express your answer as a common fraction in simplest radical form.

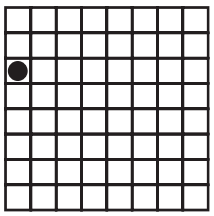
6. _____  Billy can row upstream from point A to point B in three hours. Rowing at the same rate Billy needs only one hour to row from B to A. What is the ratio of the rate of the current to the rate of Billy's rowing? Express your answer as a common fraction.

7. _____ A sequence begins 1, 4, 8, 9, 16, … and consists of all the squares and cubes of positive integers written in ascending order. Numbers that are both squares and cubes, such as 1, are written only once. What is the 50th number in this sequence?

8. _____ Starting with the number 20, a list of increasing integers – not necessarily consecutive integers – has a product that is a perfect square. What is the least possible value for the last integer in the list?

9. _____ pairs Kendra starts counting with a and counts by d, where a and d are both positive integers. For example, for $a = 5$ and $d = 3$ the sequence would be 5, 8, 11, …. The sum of two terms in Kendra's sequence is 10. How many different possible pairs (a, d) are there?

10. _____ paths On an 8 by 8 grid of unit squares, a red marker starts in the unit square called (3, 1), which is the unit square in the third row and first column. Each separate move of the red marker is to a neighboring unit square horizontally, vertically or diagonally. How many paths of exactly four moves are there from (3, 1) to (4, 5)?



Problem #3 is from the 2007 National Competition Sprint Round.