Workout 8

1%	The gravitational force on an asteroid varies inversely with the square of its distance from the sun. By what percent must the distance decrease in order that the gravitational force be multiplied by 3? Express your answer to the nearest tenth.
2. sq units	Four unit squares contain some non-shaded circles, as shown. What is the total area of the shaded regions inside the squares? Express your answer in terms of π .
3	If a represents πa^4 , then the volume of a sphere of radius 3 units could be represented by x cu units for some positive value of x . What is the value of x ? Express your answer in simplest radical form.
4	What is the value of x that satisfies the equation $2^{2 \cdot 0 \cdot 0 \cdot 8 } = 2^7 \cdot (2^7)^x$?
5	Consider all of the positive five-digit integers that can be formed using each of the digits $3, 4, 5, 6$ and 7 exactly once . What is the sum of these integers?
6	Let T be a positive integer whose only digits are 0s and 1s. If $X = T \div 12$, and X is an integer, what is the smallest possible value of X?
7angles	What is the maximum number of acute interior angles a convex pentagon can have?
8	A "deletable prime" is a positive integer that (1) is prime and (2) is either a one-digit integer or, after removing one digit, results in another deletable prime. For example, 439 is deletable because 439 is prime and deleting the 9 results in another deletable prime, 43, which is deletable because removing the 4 results in the prime 3. What is the smallest deletable prime larger than 443?
9	What is the least positive common fraction that is an integral multiple of $\frac{9}{28}$, $\frac{12}{35}$ and $\frac{15}{56}$?
10%	A large tank contains a 400-kg mixture of water and alcohol. The mixture is 64% alcohol by weight. At each step, 100 kg of the mixture will be drained from the tank, replaced with 100 kg of water, and then stirred. After three steps, what percent of the final solution will be alcohol?

Problem #6 is from the 2007 National Competition Target Round.

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