MathCounts deals with the quick manipulation of numbers. It is important that you learn the meaning of various key words which MathCounts uses.

Positive Integers: The **natural numbers**: 1, 2, 3, 4, 5, 6, . . .

Zero: 0

Negative Integers: -1, -2, -3, -4, -5, -6, -7, . . .

All Integers: . . . , -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, . . .

Addition: Any two integers can be added to give another integer.
The order of addition does not matter.

Ex: $2 + 3 = 3 + 2 = 5$, $7 + 12 = 12 + 7 = 19$, $(-3) + 19 = 19 + (-3) = 16$, $(-4) + (-12) = -16$, $(\text{anything}) + 0 = (\text{anything})$, $(anything) + (-anything) = 0$.

Subtraction: Subtraction is the same as the addition of a negative number.

Ex: $2 - 3 = 2 + (-3) = (-3) + 2 = -1$, $3 - (-5) = 3 + (-(-5)) = 3 + 5 = 8$, $-3 - 5 = -3 + (-5) = -8$, $(anything) - (anything) = (anything) + (-anything) = 0$.

Multiplication: $3 \cdot 5$ means $5 + 5 + 5 = 15$ (add 5 three times). Since $3 \cdot 5 = 5 \cdot 3$, $3 \cdot 5$ also means $3 + 3 + 3 + 3 + 3 = 15$ (add 3 five times).

Ex: $(-1) \cdot (anything) = (-anything)$, $(-1) \cdot (-5) = 5$, $17 \cdot (-20) = -340$.

Multiples: $156 = 1 \cdot 12$ so 156 is a **multiple** of 12 (156 is also a **multiple** of 13).

Even Integers: Any integer which is a multiple of 2 is called an **even integer**.
Ex: . . . , -6, -4, -2, 0, 2, 4, 6, . . .

Odd Integers: Any integer which is not an even integer.
Ex: . . . , -7, -5, -1, 1, 3, 5, . . .

Division: $15 \div 3 = 5$ since $15 = 3 \cdot 5$, $(-15) \div 3 = -5$ since $-15 = 3 \cdot (-5)$, $(-15) \div (-3) = 5$ since $-15 = (-3) \cdot 5$.

Factors: Factors of an integer exactly divide the integer.
Ex: 1, 3, 5, 15 are all factors of 15. 1, 2, 4, 5, 10, 20 are factors of 20.

Primes: A **prime** number is any integer **greater than 1** which is divisible only by itself and 1.
Ex: 15 is not prime since it is divisible by 5. 1 is **not** a prime.

Positive Exponent: The number of times a number is multiplied by itself. Exponents are also called **powers**.
Ex: $7^4 = 7 \cdot 7 \cdot 7 \cdot 7$, $\pi^5 = \pi \cdot \pi \cdot \pi \cdot \pi \cdot \pi$, $6^2 = 36$.

Rules Of Exponents: Same number, different exponents: $(6^3) \cdot (6^5) = 6^{3+5} = 6^8$.

Same exponent, different numbers: $(3^4) \cdot (5^4) = (3 \cdot 5)^4 = 15^4$.

Power of a power: $(21^7)^5 = 21^{7 \bullet 5} = 21^{35}$.

Quick Divisibility: It is important to be able to factor integers quickly and efficiently.

2 is a factor of an integer if 2 divides the last digit of the integer.
      <u>Ex:</u> 2 divides 7936 because 2 divides 6.

4 is a factor of an integer if 4 divides the last two digits of the integer.
      <u>Ex:</u> 4 divides 7936 because 4 divides 36.

8 is a factor of an integer if 8 divides the last three digits of the integer.
      <u>Ex:</u> 8 divides 7936 because 8 divides 936.

3 is a factor of an integer if 3 divides the sum of the digits of the integer.
      <u>Ex:</u> 3 divides 7935 because 3 divides $7+9+3+5 = 24$.
        (3 divides 24 because 3 divides $2+4 = 6$.)

9 is a factor of an integer if 9 divides the sum of the digits of the integer.
      <u>Ex:</u> 9 divides 7938 because 3 divides $7+9+3+8 = 27$.
        (9 divides 27 because 9 divides $2+7 = 9$.)

5 is a factor of any integer ending in 0 or 5.
      <u>Ex:</u> 5 divides 7935 because it ends in 5. 5 divides 793560 because it ends in 0.

10 is a factor of any integer ending in 0.
      <u>Ex:</u> 10 divides 793560 because it ends in 0.

11 is a factor of an integer if 11 divides the **alternating sum** of the digits of the integer.
      <u>Ex:</u> 11 divides 71929 because 11 divides $7-1+9-2+9 = 22$.
        (11 divides 22 because 11 divides $2-2 = 0$.)

Homework Problems

(NO CALCULATORS)

a) $-347 + \left(-673\right)$ b) $-329 - \left(-462\right) - 71$

c) How many positive integers are greater than –20 and less than 30 ?

d) How many integers are greater than –15 and less than 17 ?

e) How many positive factors does 21 have?

f) How many prime factors does 21945 have?

g) What is 21945 divided by 1155 ? Hint: first, factor both numbers!!

h) What is $2^7 \bullet 5^7$?

i) What is 6400 divided by 5^2 ?

j) What is 6400 divided by 4^4 ?