

Fractions: A **fraction** is a ratio of two integers. The horizontal line indicates division.

Ex: $\frac{7}{9}$, $\frac{543}{172}$, $\frac{-16}{12}$, $\frac{16}{-12}$, $\frac{-43}{-67}$ are all fractions. $\frac{7}{9}$ means 7 divided by 9.

Numerator: That part of a fraction **above** the line.

Denominator: That part of a fraction **below** the line.

Ex: In $\frac{7}{9}$ the numerator is 7 and the denominator is 9.

Simplifying Fractions: Fractions may be **simplified** by dividing out the same factors in both numerator and denominator.

A **completely simplified** fraction is a fraction whose numerator and denominator have no common factors (except 1), and the denominator is a positive integer.

Unless the problem states otherwise, **all fractional answers must be in completely simplified form.**

Ex: $\frac{16}{12} = \frac{4 \cdot 4}{4 \cdot 3} = \frac{4}{3}$, $\frac{-24}{-6} = \frac{(-1)(24)}{(-1)(6)} = \frac{24}{6} = \frac{4 \cdot 6}{1 \cdot 6} = \frac{4}{1} = 4$.

Decimals: A **decimal** is a shorthand way of writing an integer divided by a power of 10.

The number of digits to the right of the decimal point equals the power of 10.

Ex: $0.234 = \frac{234}{10^3} = \frac{234}{1000}$, $1.37 = \frac{137}{10^2} = \frac{137}{100}$, $17.9 = \frac{179}{10^1} = \frac{179}{10}$, $0.0013 = \frac{13}{10^4} = \frac{13}{10000}$.

Terminating Decimal: Any decimal number which has **only zeros** to the **right** of a certain digit.

These zeros are not usually written.

Ex: $0.234 = 0.234000000\dots$, $17.9 = 17.90000\dots$

Repeating Decimal: Any decimal number which is **not** terminating and which **repeats** some collection of digits.

A repeating decimal is written with a bar over the repeating digits.

Ex: 0.234234234 is terminating and so is **not** repeating.

$0.\overline{234} = 0.234234234\dots$ is a repeating decimal.

Those three little dots make a **big** difference!

Decimal / Fraction: Any repeating or terminating decimal can be written as a fraction.

Ex: $0.\overline{234} = \frac{234}{999}$, $1.\overline{734} = 1.3 + 0.\overline{43} = \frac{13}{10} + \frac{43}{99}$.

The number of 9's in the denominator equals the number of repeating digits.

Sometimes you have to be clever:

$0.7924\overline{7} = 0.792 + 0.0004\overline{7} = \frac{792}{10^3} + \frac{0.4\overline{7}}{10^3} = \frac{792}{1000} + \frac{47/99}{1000} = \frac{792}{1000} + \frac{47}{99000}$.

Any fraction can be written as a repeating or terminating decimal.

Ex: $\frac{2}{3} = \frac{6}{9} = 0.\overline{6}$, $\frac{2}{11} = \frac{18}{99} = 0.\overline{18}$, $\frac{1}{4} = 0.25$.

The only fractions in completely simplified form which can be written as terminating decimals are those whose denominators can be factored into products of powers of 2 and powers of 5.

Ex: $\frac{1}{4} = \frac{1}{2^2} = \frac{5^2}{2^2 \cdot 5^2} = \frac{25}{10^2} = 0.25$, $\frac{3}{20} = \frac{3}{10 \cdot 2} = \frac{3 \cdot 5}{10 \cdot 2 \cdot 5} = \frac{15}{10^2} = 0.15$.

Ex: $\frac{19}{350} = \frac{19}{7 \cdot 50}$ does **not** terminate (all you need do is find one factor which is not a power of 5 or 2, in this case 7).

NUMBERS 2

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Rounding: It is often necessary to **round** decimals to a certain number of **significant digits**. To do this, count the required number of digits from the left starting with the **first non-zero digit**.

Then: Rule: If the next digit is 0, 1, 2, 3, 4 then replace the remaining digits with zeros.

If the next digit is 5, 6, 7, 8, 9 then increase the last significant digit by 1 and replace the remaining digits with zeros.

Ex: Round the following numbers to three significant digits: 7916, 7914, 16.437 .

Soln: In each case count three digits from the left and examine the fourth digit:

7916, so increase 1 by 1 and replace the remaining digits with zeros to get 7920.

7914, so replace remaining digits with zeros to get 7910.

16.437, so replace remaining digits with zeros to get 16.400 = 16.4 .

Ex: What is 79.896 to four significant figures?

Soln: Count four digits from the left and examine the fifth digit: 79.896 .

Increase 89 by 1 to get 79.900 , so the answer is 79.90 . Notice that a zero is kept to emphasize that the number is accurate to four significant digits. 79.9 is accurate to only three significant digits.

Ex: What is 0.079896 to four significant figures?

Soln: Count four digits from the left starting with the first non-zero digit: 0.079896 .

Increase 89 by 1 to get 0.07990 . As above, the last zero must be kept.

Numbers 2 Homework Problems (NO CALCULATORS)

- Write the simplest common fraction which names the same number as $0.\overline{3}$.
- Write the simplest common fraction which names the same number as $0.8\overline{3}$.
- Write the simplest common fraction which names the same number as $0.\overline{24}$.
- Write the common fraction equivalent to $0.5\overline{7}$.
- What is the fraction equivalent to $2.0\overline{45}$?
- Write as a decimal: $\frac{5}{16}$.
- Write the simplest common fraction which names the same number as $0.\overline{36}$.
- Give the letter(s) corresponding to the fraction(s) given which, when written as decimals, will not terminate. (a) $\frac{3}{150}$, (b) $\frac{7}{75}$, (c) $\frac{11}{250}$, (d) $\frac{15}{48}$.
- To what decimal is $\frac{1}{40}$ equivalent?
- Express $\frac{21}{11}$ as a repeating decimal.
- Express $0.\overline{164}$ as a common fraction.

- l) What is $0.\overline{49} - 0.\overline{35}$? Express your answer as a common fraction.
- m) Express 0.0138 as a common fraction.
- n) Express $\frac{1}{30}$ as a decimal.
- o) Express $0.\overline{26}$ as a common fraction.
- p) Express $0.\overline{25} / 0.\overline{5}$ as a common fraction.
- q) The fraction $\frac{13}{(128)(125)}$ is equivalent to what terminating decimal?
- r) What is the repeating decimal equivalent to $\frac{118}{55}$?
- s) What is 127,986 to three significant figures?
- t) What is 127,986 to two significant figures?
- u) What is 127,986 to four significant figures?
- v) What is 127,986 to five significant figures?
- w) What is 12.7986 to three significant figures?
- x) What is 12.7986 to two significant figures?
- y) What is 12.7986 to four significant figures?
- z) What is 12.7986 to five significant figures?
- aa) What is 0.000127986 to four significant figures?