Sums of Integers:

The sum of an Odd number of Odd integers is Odd.
The sum of an Even number of Odd integers is Even.
The sum of Any number of Even integers is Even.

Products of Integers:

The product of two Odd integers is Odd.
The product of two Even integers is Even.
The product of an Even integer with an Odd integer is Even.

Number Base: Any number is actually a sum of powers of its base.

$4035_{(7)}$ denotes a number to base 7:

$4035_{(7)} = 4 \bullet 7^3 + 0 \bullet 7^2 + 3 \bullet 7^1 + 5 \bullet 7^0 = 4 \bullet 7^3 + 3 \bullet 7 + 5$.

$4035.26_{(7)} = 4 \times 7^3 + 0 \times 7^2 + 3 \times 7^1 + 5 \times 7^0 + 2 \times 7^{-1} + 6 \times 7^{-2} = 4 \times 7^3 + 3 \times 7 + 5 + \frac{2}{7} + \frac{6}{7^2}$.

An integer in an odd base is odd if it contains an odd number of odd digits. Otherwise it is even. An integer in an even base is even or odd if the last digit is even or odd.

Ex: Is $4135_{(7)}$ an even or odd number?

Soln: 7 is an odd integer. Therefore any integer power of 7 is an odd integer. Therefore each term in the base expansion is even or odd depending as the digit is even or odd. Therefore the number is even or odd depending as the number of odd digits in the number is even or odd. $4135_{(7)}$ contains three (an odd number) odd digits (1,3,5).

Therefore $4135_{(7)}$ is **odd**.

Ex: Is $1354_{(7)}$ an even or odd number? Is $4136_{(7)}$ an even or odd number?

Soln: Same steps as above show that $1354_{(7)}$ is **odd** and that $4136_{(7)}$ is **even**.

Greatest Common Divisor (GCD): The **greatest common divisor** of several positive integers is the **largest** number which divides **all** the integers.

Least Common Multiple (LCM): The **least common multiple** of several positive integers is the **smallest** number which is divisible by **all** the integers.

Find the GCD or LCM of several integers in three steps.

1) Write the prime decomposition of each integer.
2) Rewrite these so that if a prime appears as a factor for one of the integers then it appears as a factor in all of the integers, using 0 powers where necessary.
3) Write the product of all the different primes each raised to a certain power.

   GCD = (product) where the power of each prime is chosen to be the **smallest** power of that prime which appears in step 2 .

   LCM = (product) where the power of each prime is chosen to be the **largest** power of that prime which appears in step 2 .

Ex: Find the GCD and LCM of 72, 30, and 40.

Soln: $72 = 2^3 \bullet 3^2 \bullet 5^0$, $30 = 2^1 \bullet 3^1 \bullet 5^1$, $40 = 2^3 \bullet 3^0 \bullet 5^1$.

$GCD = 2^1 \bullet 3^0 \bullet 5^0 = 2$, and $LCM = 2^3 \bullet 3^2 \bullet 5^1 = 360$.

Common Bases:

Base 10, or **decimal**: Used in everyday life.
Base 2, or **binary**: Used in computers; represents turning a switch **on** (1) or **off** (0).
In computers one binary digit is called a **bit**.
Base 8, or **octal**: Used to represent large binary numbers; easy conversion to base 2 .
In computers 8 bits is called a **byte**.
Base 16, or **hexadecimal**: Used to represent large binary numbers:
0, 1, . . . , 8, 9, A, B, C, D, E, F ; easy conversion to base 2 .
In computers 16 bits (2 bytes) is called a **word**. (Words can be larger than 2 bytes.)

Ex: Convert 7, 6, 12 to binary.

Soln: $7 = 4 + 2 + 1 = 1\times 2^2 + 1\times 2^1 + 1\times 2^0 = 111_{(2)}$.

$6 = 4 + 2 = 1\times 2^2 + 1\times 2^1 + 0\times 2^0 = 110_{(2)}$.

$12 = 8 + 4 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 = 1100_{(2)}$.

Ex: Convert octal 63 to binary. Convert hexadecimal C5 to binary.

Soln: Convert each digit to binary in triplets, then collect digits:

$6 = 110_{(2)}$, and $3 = 011_{(2)}$, so $63_{(8)} = 110011_{(2)}$.

Convert each digit to binary in quadruplets, then collect digits:

$C_{(16)} = 8 + 4 = 1100_{(2)}$, and $5_{(16)} = 4 + 1 = 0101_{(2)}$, so $C5_{(16)} = 11000101_{(2)}$.

Ex: Convert binary 110011 to octal. Convert 11000101 to hexadecimal.

Soln: Separate digits into triplets starting from **right**. Convert each triplet, then collect digits: $110011_{(2)} = 110,011_{(2)} = 6_{(8)},3_{(8)} = 63_{(8)}$.

Separate digits into quadruplets starting from **right**. Convert each quadruplet, then collect digits: $11000101_{(2)} = 1100,0101_{(2)} = C_{(16)},5_{(16)} = C5_{(16)}$.

Numbers 4 Homework Problems

(NO CALCULATORS)

a) What is the decimal value of $1234_{(6)}$?

b) What is the octal value of $11001_{(2)}$?

c) What is the binary value of $17_{(10)}$?

d) What is the hexadecimal value of $17_{(10)}$?

e) What is the hexadecimal value of $17_{(8)}$?

f) What is the binary value of $17_{(8)}$?

g) What is the hexadecimal value of the binary number 1111 ?

h) What is the octal value of the binary number 1010110011100011 ?

i) What is the hexadecimal value of the binary number 1010110011100011 ?

j) What are the GCD and LCM of 48 and 72 ?

k) What are the GCD and LCM of 48, 72, and 120 ?

l) Is $1234567890_{(17)}$ even or odd?

m) What is the decimal integer ratio of the GCD and LCM of $74_{(8)}$ and $120_{(8)}$?

n) Two gears are meshed together. One gear has 24 teeth and the other gear has 60 teeth. If the gears start turning how many complete revolutions must the 24 tooth gear make before both gears return to their original positions?

o) What is (1,800,000)/(60,000) ? Express your answer as an integer.

p) What is (1,800,000)/(0.0006) ? Express your answer in scientific notation.

q) Write the common fraction equivalent to $0.0\overline{351}$.

r) 17 is what percent of 51 ? Express your answer to the nearest tenth of a percent.

s) Find the positive value of the difference between the two largest of the following numbers. Express your answer as a decimal. (a) GCD (24,60,120), (b) LCM (24,60,120), (c) GCD (32,80,160), (d) LCM (32,80,160), (e) LCM (4,5,9) .

t) What is $(0.0000000003)(1,700,000,000,000)$?

u) What is $\frac{(80,000,000)^2(0.000006)^3}{(600,000)^3(0.0002)^4}$?

v) Which pair(s) are relatively prime: (a) (23,115) , (b) (72,35) , (c) (256,27) , (d) (990,637) ?