

Ordering: An **inequality** is a statement that one quantity or expression is greater or less than another quantity or expression. Relevant symbols and their meanings are as follows:

$a > b$ means **a is greater than b** ($a - b$ is a positive number);

$a < b$ means **a is less than b** ($a - b$ is a negative number);

$a \geq b$ means **a is greater than or equal to b** ;

$a \leq b$ means **a is less than or equal to b** ;

Ex: $-4 < b \leq 17$ means **b is greater than -4 and less than or equal to 17** .

Rules: If $a \geq b$ then $a + c \geq b + c$ for any number c . Similarly for $>$, $<$, and \leq .

If $a \geq b$ and $k > 0$ then $ka \geq kb$ for any **positive** number k . Similarly for $>$, $<$, and \leq .

If $a \geq b$ and $k < 0$ then $ka \leq kb$ for any **negative** number k . Similarly for $>$, $<$, and \leq .

If $a \geq b$ and a, b, n are **positive** then $a^n \geq b^n$ but $a^{-n} \leq b^{-n}$. Similarly for $>$, $<$, and \leq .

If $a \geq b$ and $c \geq d$ then $(a + c) \geq (b + d)$. Similarly for $>$, $<$, and \leq .

If $a \geq b \geq 0$ and $c \geq d \geq 0$ then $ac \geq bd$.

Ex: $7 > 6$ and $6 > 0$ with $2 > 0$ means that $7^2 > 6^2$ but $7^{-2} < 6^{-2}$.

Absolute Value: The **absolute value** of a number is the positive part of the number. The symbol for absolute value is a vertical line on each side of the number: $|-17| = 17$, $|17| = 17$.

Rules of Fractions:

Multiplication: Multiply numerators together and denominators together: $\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$.

Division: Invert divisor and then multiply: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$. $\frac{a/b}{c/d} = \frac{a}{b} \bullet \frac{d}{c} = \frac{ad}{bc}$.

Addition: Fractions can be added only when they have the same denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{d} + \frac{c}{d} \bullet \frac{b}{b} = \frac{ad}{bd} + \frac{cb}{db} = \frac{ad+cb}{bd} .$$

Subtraction: Add the negative fraction: $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left(-\frac{c}{d}\right) = \frac{a}{b} + \frac{(-c)}{d} = \frac{ad+b(-c)}{bd} = \frac{ad-bc}{bd}$.

Ex: $\frac{7}{11} \bullet \frac{8}{9} = \frac{7 \bullet 8}{11 \bullet 9} = \frac{56}{99}$.

Ex: $\frac{2/3}{5/7} = \frac{2}{3} \bullet \frac{7}{5} = \frac{2 \bullet 7}{3 \bullet 5} = \frac{14}{15}$.

Ex: $\frac{1}{3} + \frac{1}{12} = \frac{4}{12} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$. Another way: $\frac{1}{3} + \frac{1}{12} = \frac{1}{12} \left(\frac{12}{3} + 1\right) = \frac{1}{12} (4 + 1) = \frac{5}{12}$.

Ex: $\frac{1}{3} - \frac{4}{7} = \frac{7}{3 \bullet 7} - \frac{4 \bullet 3}{7 \bullet 3} = \frac{7-12}{21} = -\frac{5}{21}$. Another way: $\frac{1}{3} - \frac{4}{7} = \frac{1}{7} \left(\frac{7}{3} - 4\right) = \frac{1}{7} \left(\frac{1}{3}\right) (7 - 4 \bullet 3) = -\frac{5}{21}$.

Fraction Definitions:

Mixed Number: An integer plus a fraction between 0 and 1.

Ex: $3\frac{1}{2}$, $2\frac{1}{3}$, $-17\frac{43}{97}$.

Common Fraction: A fraction written with **NO** integer part.

Ex: $\frac{7}{2}$, $\frac{14}{6}$, $-\frac{327}{109}$.

Simplest Form: A fraction written with no common divisor of numerator and denominator.

Unless stated otherwise, fractional answers must always be in simplest form.

Ex: $\frac{7}{2}$, $\frac{7}{3}$, -3 , $-14\frac{2}{3}$.

Reciprocal: The **reciprocal** of a number equals 1 divided by the number.

Ex: Reciprocal of 2 equals $\frac{1}{2}$.

Reciprocal of $\frac{3}{2}$ equals $\frac{1}{3/2} = 1 \cdot \frac{2}{3} = \frac{2}{3}$.

To multiply mixed numbers, first convert each mixed number to a common fraction, then multiply.

To divide mixed numbers, first convert each mixed number to a common fraction, then divide.

Ex: $2\frac{3}{4} \times 3\frac{1}{2} = \frac{11}{4} \times \frac{7}{2} = \frac{11 \cdot 7}{4 \cdot 2} = \frac{77}{8} = 9\frac{5}{8}$.

$2\frac{3}{4} \div 3\frac{1}{2} = \frac{11}{4} \div \frac{7}{2} = \frac{11}{4} \times \frac{2}{7} = \frac{11 \cdot 2}{4 \cdot 7} = \frac{11}{2 \cdot 7} = \frac{11}{14}$.

Ratio: A **ratio** of two numbers is a fraction, and is usually written with two colons between the numbers.

Ex: The ratio of dogs to cats is 4::5 . If there are 40 cats how many dogs are there?

Soln: Number of dogs = $\frac{4}{5} \times 40 = 32$.

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Soln: Number of dogs = $\frac{5}{4} \times 40 = 50$.

Proportion: A **proportion** is a ratio.

Ex: The number of dogs is in proportion to the number of cats as 4::5 . If there are 40 cats how many dogs are there?

Soln: Number of dogs = $\frac{4}{5} \times 40 = 32$.

Ex: The number of cats is in proportion to the number of dogs as 4::5 . If there are 40 cats how many dogs are there?

Soln: Number of dogs = $\frac{5}{4} \times 40 = 50$.

Numbers 5 Homework Problems

(NO CALCULATORS)

- Express the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ as a common fraction in simplest form.
- Divide $3\frac{1}{2}$ by $2\frac{1}{3}$. Express your answer as a common fraction in simplest form.
- What is $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$, expressed as a mixed number in simplest form?
- Express as a mixed number: $15 - (3\frac{1}{8} + 6\frac{5}{8})$.
- Express as a common fraction: $17\frac{2}{3} - 21\frac{4}{7} + 3\frac{3}{4} + 1\frac{4}{5}$.
- Express $\frac{\frac{1}{2} + \frac{1}{3} - \frac{3}{4} + \frac{1}{5}}{\frac{1}{2} - \frac{1}{3} + \frac{3}{4} - \frac{1}{5}}$ as a common fraction.

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g) 2.5 is what percent of 40 ? Express your answer as a decimal to three significant figures.

h) Express $12\frac{3}{4}\%$ as a common fraction.

i) Express 10% of 30% of 50 as a decimal.

j) Express the reciprocal of 2.75 as a common fraction.

k) Express the product $(0.4)(0.4)$ as a common fraction.

l) What is $\frac{3^8}{3^5} - \frac{4^2 \cdot 2^4}{2^6} + 3(-2)^3 + \frac{2(-3)^2}{6}$?

m) Express as a common fraction $\frac{5}{9} - \frac{2}{3}$.

n) Express as a common fraction $\frac{8}{3} \div \frac{3}{4}$.

o) Express as a common fraction $\frac{3(-3)^2 + 4(-2)^3}{2^3 - 3^2}$.

p) Express as a common fraction $\frac{5^7}{5^4} + \frac{2^{10}}{8^2(-2)^3} - 4(-3)^4$.

q) Express as a common fraction $\left(\frac{2^{20}}{3^{15}}\right)^{1/5}$.

r) Express as a common fraction $(6^{20})^{1/10}$.

s) Express as a decimal to three significant figures $(81)^{5/4}$.

Evaluate the following:

t) $\left|2 - \frac{14}{3}\right|$

u) $\left|-\frac{3}{2}\right|$

v) $|-3.14|$

w) $|0|$

x) $\left|\frac{5}{3} - 2\frac{1}{9}\right|$