**Rules of Exponents:** $a^n = a \bullet a \bullet a \bullet ... \bullet a$ ($n$ factors $a$).

$a$ is called the **base** while $n$ is called the **exponent** and the result is called the **power**.

Rules for manipulating exponents are: $a^0 = 1$. $a^1 = a$. $a^{-n} = \frac{1}{a^n}$. $(a \bullet b)^n = a^n \bullet b^n$.

$a^n \bullet a^m = a^{n+m}$. $(a^n)^m = a^{n \bullet m}$.

**Fractional Exponents:** $a^{1/n}$ means that $(a^{1/n})^n = a$. $a^{1/n}$ is called the **nth root** of $a$.

$a^{1/2} = \sqrt{a}$. $a^{1/3} = \sqrt[3]{a}$. $a^{1/n} = \sqrt[n]{a}$. $a^{m/n} = (a^{1/n})^m$.

Special values: $2^5 = 32$, $3^5 = 243$, $2^{10} = 1024$, $(-1)^n = \pm 1$ as n is even or odd.

This means that $(32)^{1/5} = 2$, $(243)^{1/5} = 3$, $(1024)^{1/10} = 2$.

$0^0$ is not defined. If you ever get $0^0$ then you have goofed.

Exponents can have any values, not just integers or fractions.

Learn to use your calculator to find any power of any number.

(**MEMORIZE THESE RULES!!**)

**Rational Numbers:** A **rational number** is any number which can be written as a ratio of two integers.

Any **rational number** can be written as a repeating (or terminating) decimal.

Any repeating (or terminating) decimal is a rational number.

**Irrational Numbers:** An **irrational number** is any number which cannot be written as a ratio of two integers.

An **irrational number** cannot be written as a repeating (or terminating) decimal.

Ex: Show that $\sqrt{2}$ is an irrational number.

Soln: Assume that $\sqrt{2}$ is not an irrational number. Then $\sqrt{2}$ is a rational number.

Then we can write $\sqrt{2}$ as a ratio of two integers: $\sqrt{2} = a / b$ where a and b are integers **having no common factor** (otherwise we divide out the common factor).

Square both sides and multiply by $b^2$ to get $a^2 = 2b^2$.

Then $a^2$ must be an even number, so a must be an even number, say $a = 2n$.

Then $4n^2 = 2b^2$ so $b^2 = 2n^2$, so b must be an even number, say $b = 2m$.

But this means that a and b have a common factor (2) contrary to what we assumed.

Therefore we cannot write $\sqrt{2}$ as a ratio of two integers.

Therefore $\sqrt{2}$ cannot be a rational number.

Therefore $\sqrt{2}$ must be an irrational number.

(This means that $\sqrt{2}$ cannot be written as a repeating decimal.)

Ex: In fact, if P is a prime number then $\sqrt{P}$ is an irrational number.

(This means that there are lots of irrational numbers. In fact there are far, far more irrational numbers than there are rational numbers.)

**Rationalizing Denominators:** One often has to deal with strange looking denominators such as $\frac{9}{3+\sqrt{2}}$. Such denominators can be changed into rational denominators by using

$$(a+b)(a-b)=a^2-b^2 \qquad \textbf{(memorize)}$$

Ex: $\frac{9}{3+\sqrt{2}}=\frac{9}{3+\sqrt{2}}\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right)=\frac{27-9\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2}=\frac{27-9\sqrt{2}}{9-2}=\frac{27-9\sqrt{2}}{7}=\frac{27}{7}-\frac{9}{7}\sqrt{2}$.

Ex: $\frac{2}{\sqrt{17}}=\left(\frac{\sqrt{17}}{\sqrt{17}}\right)\frac{2}{\sqrt{17}}=\frac{2\sqrt{17}}{17}$.

Ex: $\frac{2}{\sqrt[7]{17}}=\frac{2}{17^{1/7}}=\left(\frac{17^{6/7}}{17^{6/7}}\right)\frac{2}{17^{1/7}}=\frac{2}{17}\sqrt[7]{17^6}$.

This process of turning a wrong form denominator into a more acceptable form is called **rationalizing the denominator** and is frequently used in all levels of mathematics.

## Numbers 6 PROBLEMS, Due Next Week

## (NO CALCULATORS)

a) Show that $\sqrt{P}$ is an irrational number? (for top students only!!)

b) Is $\frac{1}{2-\sqrt{2}}$ a rational number? Why?

c) Evaluate $\sqrt[3]{\frac{(0.004)^4(0.0036)}{(120,000)^2}}$ .

d) Simplify: $\sqrt{196}$ .

e) We know that $\sqrt[3]{8}=2$ since $2^3=8$. Compute $\sqrt[3]{1728}$ .

f) Round to the nearest whole number: $\sqrt{78}$ .

g) To which integer is $\sqrt{133}$ closest?

h) Simplify: $4\sqrt{2}\sqrt{18}$ .

i) Express in simplest form: $\sqrt{12\frac{1}{4}}$ .

j) Give the letter corresponding to the largest number. (a) $2^{100}$, (b) $3^{75}$, (c) $5^{50}$.

k) Express in simplest form: $\sqrt{3^5+3^5+3^5}$ .

l) Simplify: $256^{0.25}$ .

m) Simplify: $\sqrt{18}-\sqrt{8}+\sqrt{\frac{1}{2}}$ .

n) What is the simplest radical form of $\sqrt[3]{4}\div\sqrt[6]{2}$ .

o) Find $\sqrt{\sqrt{2{,}560{,}000}}$ .

p) Simplify: $\sqrt{128}+\sqrt{72}$ .

q) Simplify: $\sqrt{2\frac{7}{9}}$ .

r) Express in simplest form: $\sqrt{6}\times\sqrt{15}\times\sqrt{10}$ .

s) Express in simplest form: $\sqrt[4]{81}\bullet\sqrt{81}$ .

**Evaluate:**

t) $\left(\frac{3}{4}\right)^{-3}$

u) $(0.02)^{-2}$

v) $\left(\frac{64}{27}\right)^{2/3}$

w) $\left(-37^3\right)^{1/3}$

x) $-(-1)^{-17/23}$

y) $17^{13}/17^{14}$

z) $4^3\bullet 17^0$

aa) $9^{2.6}\bullet 9^{-1.8}/9^{-0.2}$

ab) $\frac{3^0-3^{-2}}{3^1-3\left(3^{-2}\right)^2}$

ac) $12^{3.1}\left(3^{4.3}\right)\left(12^{-1.6}\right)\bullet 3^{1.4}/3^{4.2}$

ad) $\frac{(-3)^2(-2x)^{-3}}{(x+1)^{-2}}$ when $x=2$

ae) $\frac{2+2^{-1}}{5}+(-8)^0-4^{3/2}$

af) $4x^{-2/3}+3x^{1/3}+2x^0$ when $x=8$

ag) $25^0+0.25^{1/2}-8^{1/3}\bullet 4^{-1/2}+0.027^{1/3}$