Units Digit: The **units digit** of a number is the digit in the units place of the number.

Ex: The units digit of 36,927.42 is 7 .

Digit In Units Place: The units digit of a **product** of numbers equals the units digit of the product of the units digits of the numbers.

Ex: What is the units digit of $743 \bullet 426$?

Soln: The units digit of 743 is 3, and the units digit of 426 is 6, so $3 \times 6 = 18$, so the units digit of the product is 8 .

Ex: What is the units digit of 2^{18} ?

Soln: Notice that the units digit of $2^5 = 32$ is 2. Write this as $2^5 \sim 2$.

$$2^{18} = 2^3 \bullet 2^{15} = 2^3 \left(2^5\right)^3 \sim 2^3 \left(2\right)^3 \sim 2^3 \bullet 2^3 = 2^6 = 2 \bullet 2^5 \sim 2 \bullet 2 = 4 ,$$

so the units digit of 2^{18} is 4 .

Ex: What is the units digit of 343^7 ?

Soln: The units digit is given by

$$343^7 \sim 3^7 = 3 \bullet 3^6 = 3\left(3^2\right)^3 = 3 \bullet 9^3 = 3 \bullet 9 \bullet 9^2 = 27 \bullet 81 \sim 7 \bullet 1 = 7 .$$

Number Sequence: A **number sequence** is a collection of numbers in a particular order.

Ex: $-6,\ -8,\ -3,\ -2,\ 0,\ 4,\ 3,\ 10,\ldots$ is a number sequence.

Arithmetic Sequence: An **arithmetic sequence** is a number sequence having a **constant difference** between consecutive numbers.

Ex: $-5,\ -1,\ 3,\ 7,\ 11,\ 15,\ldots$ is an arithmetic sequence since each number is 4 more than the previous number.

Ex: $\frac{11}{2}, \frac{19}{4}, 4, \frac{13}{4}, \frac{5}{2}, \frac{7}{4}, 1, \frac{1}{4}, -\frac{1}{2}, -\frac{5}{4}, -2, \ldots$ is an arithmetic sequence since each number is $\frac{3}{4}$ less than the previous number.

Geometric Sequence: A **geometric sequence** is a number sequence having a **constant ratio** between consecutive numbers.

Ex: $27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is a geometric sequence since each number is $\frac{1}{3}$ times the previous number.

Ex: $-\frac{1}{4}, 1, -4, 16, \ldots$ is a geometric sequence since each number is -4 times the previous number.

Number Series: A **number series** is the sum of a collection of numbers.

Ex: $-6-8-3-2+0+4+3+10+\ldots$ is a number series.

Arithmetic Series: An **arithmetic series** is the sum of an arithmetic sequence.

Ex: $-5-1+3+7+11+15+\ldots$ is an arithmetic series since each number in the sum is 4 more than the previous number.

Ex: $\frac{11}{2}+\frac{19}{4}+4+\frac{13}{4}+\frac{5}{2}+\frac{7}{4}+1+\frac{1}{4}-\frac{1}{2}-\frac{5}{4}-2-\ldots$ is an arithmetic series since each number in the sum is $\frac{3}{4}$ less than the previous number.

Sum Of An Arithmetic Series: The sum of the first n terms of an **arithmetic series** equals $\boxed{A_n = a_1 + a_2 + a_3 + \ldots + a_n = \frac{1}{2}n\left(a_1 + a_n\right)}$ where a_1 is the first term of the arithmetic series and $\boxed{a_n = a_1 + \left(n-1\right)d}$ is the nth term of the arithmetic series. d is the difference between each consecutive pair of terms.

Sum of first N integers: $1+2+3+4+\ldots+N = \frac{1}{2}N\left(N+1\right)$.

Ex: What is the sum of the series $\frac{11}{2}+\frac{19}{4}+4+\frac{13}{4}+\frac{5}{2}+\frac{7}{4}+1+\frac{1}{4}-\frac{1}{2}-\frac{5}{4}$?

Soln: $n = 10$, $d = -\frac{3}{4}$, $a_1 = \frac{11}{2}$, $a_{10} = -\frac{5}{4}$, so $A_{10} = \frac{1}{2}\bullet 10\left(\frac{11}{2}-\frac{5}{4}\right) = 5\left(\frac{22}{4}-\frac{5}{4}\right) = 5\left(\frac{17}{4}\right) = \frac{85}{4}$.

Geometric Series: A **geometric series** is the sum of a geometric sequence.

Ex: $27+9+3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ is a geometric series since each number is $\frac{1}{3}$ times the previous number.

Ex: $-\frac{1}{4}+1-4+16$ is a geometric series since each number is -4 times the previous number.

Sum Of A Geometric Series: The sum of the first n terms of a **geometric series** equals $\boxed{G_n = g_1\left(1+r+r^2+r^3+\ldots+r^{n-1}\right) = g_1\left(\frac{1-r^n}{1-r}\right)}$ where $\boxed{r<1}$ is the ratio between each consecutive pair of terms, and g_1 is the first term in the geometric series.

Ex: What is the sum of the series $27+9+3+1+\frac{1}{3}+\frac{1}{9}$?

Soln: $n = 6$, so $G_6 = 27\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots+\frac{1}{3^5}\right)$, so $r = \frac{1}{3}$, so

$$G_6 = 27\left(\frac{1-\frac{1}{3^6}}{1-\frac{1}{3}}\right) = \frac{81}{2}\left(1-\frac{1}{3^6}\right) = \frac{1}{2}\left(81-\frac{3^4}{3^6}\right) = \frac{1}{2}\left(81-\frac{1}{3^2}\right) = \frac{1}{2}\left(81-\frac{1}{9}\right) = \frac{1}{2}\left(80\frac{8}{9}\right) = 40\frac{4}{9} .$$

Sum Of An Infinite Geometric Series: The sum of an **infinite** number of terms of a **geometric** series equals $\boxed{G_\infty = g_1\left(1+r+r^2+r^3+\ldots\right) = \frac{g_1}{1-r}}$ where $\boxed{r<1}$ is the ratio between each consecutive pair of terms. The … stands for all of the remaining (infinite) number of terms in the series.

Ex: What is the sum of the series $27+9+3+1+\frac{1}{3}+\frac{1}{9}+\ldots$?

Soln: $G_\infty = 27\left(1+\frac{1}{3}+\frac{1}{3^2}+\ldots\right)$ so $r = \frac{1}{3}$ and $G_\infty = \frac{27}{1-\frac{1}{3}} = \frac{27}{\frac{2}{3}} = \frac{3}{2}\bullet 27 = \frac{81}{2} = 40\frac{1}{2}$.

Numbers 7 Homework Problems
(NO CALCULATORS)

a) $(81)^{1/2} = 3^m$. Find m.

b) How many prime numbers less than 30 are divisible by 3 or 5 ?

c) The four digit number 374n is divisible by 18 . Find the unit digit n .

d) Determine the units digit of $5^{17} - 5$.

e) Determine the units digit of $17^{13} - 17$.

f) What is the 15^{th} term in the arithmetic sequence $-17, -14, -11, \ldots$?

g) What is the 21^{st} term in the arithmetic sequence $26, 22, 18, \ldots$?

h) What is the sum of the first 15 terms in the sequence of problem (f)?

i) What is the sum of the first 21 terms in the sequence of problem (g)?

j) What is the 6^{th} term in the geometric sequence $28, 14, \ldots$?

k) What is the 10^{th} term in the geometric sequence $2187, -729, \ldots$?

l) What is the sum of the first 6 terms of the geometric sequence of problem (j)?

m) What is the sum of the first 10 terms of the geometric sequence of problem (j)?

n) What is the sum of all of the terms of the geometric sequence of problem (j)?

o) What is the sum of all of the terms of the geometric sequence of problem (k)?

p) The sum of two consecutive integers is 127 . What is the largest integer?

q) The sum of three consecutive even integers is -198 . What is the smallest integer?

r) Express as a decimal: $0.096 \div (1.44)^{1/2}$.

s) What is the product of the digits in the sum: 41,874 + 14,676 ?

t) Express in simplest form: $\sqrt{10\frac{9}{16}}$.

u) Evaluate: $75^{-1.8}\left(3^{1.3}\right)\left(45^{-2.1}\right)(15)^{4.7}$.