Factorial: A **factorial** of a positive integer is written $n!$ and equals the product of all integers between the given integer and 1, inclusive: $n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$.

Special Cases: $1! = 1$, $0! = 1$ (this is a definition).

Ex: $3! = 3 \bullet 2 \bullet 1 = 6$, $5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 120$, $n! = n \bullet (n-1)!$, $6! = 6 \bullet 5! = 6 \bullet 120 = 720$,

$7!/5! = 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1/(5 \bullet 4 \bullet 3 \bullet 2 \bullet 1) = 7 \bullet 6 = 42$, or

$7!/5! = 7 \bullet 6!/5! = 7 \bullet 6 \bullet 5!/5! = 7 \bullet 6 = 42$.

Permutations: A **permutation** is an arrangement of all or part of a number of things in a **definite order**. The **order** makes a difference.

Ex: The permutations of three letters $\{a,b,c\}$ taken three at a time are

$\{abc, acb, cab, cba, bca, bac\}$.

Ex: The permutations of three letters $\{a,b,c\}$ taken two at a time are

$\{ab, ba, ac, ca, bc, cb\}$.

The number of different permutations of *n* things taken *r* at a time is $\boxed{{}_nP_r = P(n,r) = \frac{n!}{(n-r)!}}$.

There are n ways of choosing the first, $n-1$ ways of choosing the second, $n-2$ ways of choosing the third, . . . , $n-r+1$ ways of choosing the rth, so the product is ${}_nP_r$.

Ex: The number of ways in which 4 persons can take their places in a cab having 6 seats is

(6 seats taken 4 at a time) ${}_6P_4 = P(6,4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6 \bullet 5 \bullet 4 \bullet 3 = 18 \bullet 20 = 360$.

The number of permutations of *n* things taken *n* at a time, of which n_1 are alike, n_2 others are alike, n_3 others are alike, etc. is $\boxed{P = \frac{{}_nP_n}{(n_1!)(n_2!)(n_3!)}}$.

For each possible permutation, there are $n_1!$ ways of interchanging the n_1 identical things which do not give different results, and similarly for the other sets of identical things.

Ex: The number of ways 3 dimes and 7 quarters can be distributed among 10 boys with

one coin per boy is $P = \frac{10!/(10-10)!}{3!7!} = \frac{10 \bullet 9 \bullet 8}{3 \bullet 2 \bullet 1} = 10 \bullet 3 \bullet 4 = 10 \bullet 12 = 120$.

Ex: The number of different three-letter words which can be made from the word SEE is

$P = \frac{3!}{2!} = 3$. (The words do not have to make sense.)

The number of ways of arranging *n* different things taken *r* at a time around a circle is $\boxed{\frac{{}_nP_r}{r}}$.

For each possible arrangement, shifting all objects by 1 in the same direction does not change the order of the objects. There are *r* possible shifts which is why the total number of possible permutations is divided by *r*.

Ex: The number of different ways to seat 5 people around a circular table is

$\frac{5!/(5-5)!}{5} = 4! = 24$.

Ex: The number of different ways to seat 5 people in a straight line is $\frac{5!}{(5-5)!} = 120$.

Combinations: A **combination** is a grouping of all or part of a number of things with **no definite order**. The order does **not** make a difference.

Ex: The combinations of three letters $\{a,b,c\}$ taken three at a time are $\{abc\}$.

Ex: The combinations of three letters $\{a,b,c\}$ taken two at a time are $\{ab,ac,bc\}$.

The number of different combinations of *n* things taken *r* at a time is $_nC_r = C(n,r) = \dfrac{n!}{r!(n-r)!}$.

Ex: The number of different handshakes that may be exchanged among a group of 12 students is $_{12}C_2 = \dfrac{12!}{2!(12-2)!} = \dfrac{12!}{2!10!} = \dfrac{12 \bullet 11}{2} = 6 \bullet 11 = 66$.

The total number of combinations C of *n* different things taken 1, 2, 3, 4, …, *n* at a time is $\boxed{C = 2^n - 1}$.

Ex: A person has in her pocket a quarter, a dime, a nickel, and a penny. The total number of ways she can remove some amount of money from her pocket is $2^4 - 1 = 15$.

Numbers 8 Homework Problems (NO CALCULATORS)

a) What is $\left(6 \times \frac{1}{4}\right) \times 8$, in simplest form.

b) A gallon mixture of tea and cream contains 95% tea and 5% cream. If one quart of pure tea is added to the mixture, what percent of the new mixture is tea?

c) By how much does the expression with the greater value exceed the expression with the lesser value: $\frac{5!}{3!}$ $\quad \frac{6!}{3! \bullet 2!}$?

d) Express in simplest form: $\frac{576 \bullet 4}{0.5 \bullet 192}$.

e) Give the value of the greater expression in simplest form: $\frac{7}{9}$ of 180 or 7.5% of 2000 .

f) A box is one-third full of marbles. If one-fourth of the marbles are removed, what percent of the box is filled with marbles?

g) Express $0.\overline{36} - 0.36$ as a common fraction.

h) Write in simplest form: $4!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$.

i) Express in simplest form: $\frac{(5-(-3))(-8+(-7))}{-10+5}$.

j) Give the letter corresponding to the smallest of the following numbers:

(a) $\frac{1}{2}\bullet\frac{2}{3}\bullet\frac{3}{4}\bullet\ldots\bullet\frac{9}{10}$, (b) $\frac{1}{10^0}$, (c) 0.5 , (d) $\sqrt{0.25}$, (e) 8.74×10^{-2} .

k) Find 3^5 .

l) Find $(-3)^5$.

m) Find $\left(\frac{2}{3}\right)^4$.

n) Find $\left(\frac{1024}{243}\right)^{1/5}$.

o) Evaluate $\frac{\sqrt{72}}{\sqrt{35}}\div\frac{\sqrt{30}}{\sqrt{21}}$.

p) If ${}_nP_r = 3024$ and ${}_nC_r = 126$ what is *r*?

q) How many different sets of 4 students can be chosen out of 17 qualified students to represent a school in a mathematics contest?

r) Find *n* if $7P(n,3) = 6P(n+1,3)$.

s) Find *n* if $3P(n,4) = P(n-1,5)$.

t) How many different four-letter words can be made from the letters HAND?

u) How many different three-letter words can be made from the letters HAND?

v) How many different four-letter words can be made from the letters ONTO?

w) How many different five-letter words can be made from the letters TONTO?

x) How many different eleven-letter words can be made from the letters MISSISSIPPI?

y) Twelve different pictures are available, of which 4 are to be hung in a row. In how many ways can this be done?

z) Twelve different pictures are available, of which 4 are to be hung in a circular room. In how many ways can this be done?