Number Of factors Of An Integer: The number of possible integer factors of an integer can be found as follows. Write the integer in terms of its prime decomposition. Add 1 to each of the exponents appearing in the prime decomposition. Multiply these numbers together.

Ex: How many factors does 72 have?

Soln: $72 = 2^3 \bullet 3^2$, so the number of factors of 72 is $(3+1)(2+1) = 12$.

Any factor of 72 must divide $2^3 \bullet 3^2$, and so must be composed of products of powers of 2 and powers of 3. The only powers of 2 possible are 0, 1, 2, and 3 . The only powers of 3 possible are 0, 1, and 2 . Each possible power of 2 can be combined with any of the possible powers of 3 giving $4 \times 3 = 12$ possible factors of 72 .

Ex: How many odd factors does 72 have?

Soln: No odd factor can be divisible by 2 . Therefore there are $(1)(2+1) = 3$ odd factors.

Ex: How many even factors does 72 have?

Soln: We know the answer must be $12 - 3 = 9$ (why?). Now let us work it out. Every even factor must be divisible by 2 . Therefore the power of 2 in the prime decomposition of any even factor must be greater than 0 , giving $(3)(2+1) = 9$.

Ex: What kinds of integers have an odd number of factors?

Soln: The product $(e_1+1)(e_2+1)(e_3+1)\ldots$ must be odd, where e_1, etc. are the exponents in the prime decomposition of the integer. Therefore none of the $(e+1)$ can be even. Therefore all of the e_1, etc. must be even. Therefore the integer must be a perfect square. **Only perfect squares have an odd number of factors.**

Sum of Consecutive Integers: A common question is to say "The sum of N consecutive integers is M", and then to ask a question about the integers. To quickly find the required integers estimate M / N . The result is very close to the middle of the required string of integers.

Ex: What is the smaller of the two consecutive integers whose sum is 64,261?

$64{,}261 / 2 = 32{,}130.5$ so try 32,130 and 32,131 (these work!) so the answer is 32,130.

Another Way: Let x be the unknown integer. Then $x + (x+1) = 2x + 1 = 64{,}261$ so

$2x = 64{,}260$ so $x = 32{,}130$

Product of Consecutive Integers: A common question is to say "The product of N consecutive integers is M", and then to ask a question about the integers. To quickly find the required integers estimate $M^{1/N}$. The result is very close to the middle of the required string of integers.

Ex: What is the smaller of the two consecutive even integers whose product is 960?

$30^2 = 900$ so the integers are a bit more than 30. Try 30×32 and get 960.

BE CAREFUL! We also have $(-30) \times (-32) = 960$ so the answer is -32 .

Statistics: Let $C = \{c_1, c_2, ..., c_n\}$ be a collection of *n* numbers.

The **mode** of C is that value in C which occurs most frequently.

The **mean** or **arithmetic mean** of C equals $(c_1 + c_2 + ... + c_n)/n$.

The **geometric mean** of C equals $(c_1 \bullet c_2 \bullet ... \bullet c_n)^{1/n}$.

The **median** of C is that value of C which has as many numbers of C below it as there are above it.

For an odd number of entries in C the median is unique.

For an even number of entries in C the median is the average of the two middle numbers.

Ex: Let $C = \{3,5,7,7\}$. The mode of C is 7.

The mean of C is $(3+5+7+7)/4 = 22/4 = 5\frac{1}{2}$.

The geometric mean of C is $(3 \bullet 5 \bullet 7 \bullet 7)^{1/4} = 735^{1/4} \cong 5.2$.

The median of C is $(5+7)/2 = 6$.

Numbers 9 Homework Problems
(NO CALCULATORS)

a) How many factors does 1998 have?

b) How many odd factors does 1998 have?

c) $(81)^{1/2} = 3^m$. Find m.

d) How many prime numbers less than 30 are divisible by 3 or 5 ?

e) How many integers <u>between</u> 1 and 100 have an odd number of factors?

f) The four digit number <u>374n</u> is divisible by 18. Find the unit digit n.

g) Express as a common fraction the reciprocal of $0.\overline{7}$.

h) Express as a common fraction: $0.\overline{56} + 0.\overline{10} - 0.\overline{3}$.

i) The sum of three consecutive odd integers is 345. What is the value of the largest integer?

j) A book is opened and the product of the two page numbers that appear is 3906. What is the sum of the two page numbers?

k) Simplify: $\frac{7!}{3!4!}$

l) If $15! = n(12!)$, find n.

m) A data set of three positive integers has a mean of 10 and a median of 6. Find the largest possible member of this set.

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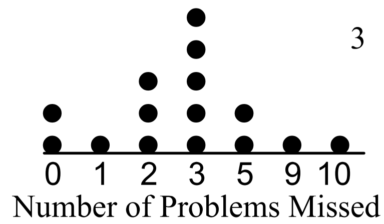
n) Use the histogram to determine the mean number of problems missed on the last mathematics test. Express your answer as a decimal. Each • represents two (2) students. What is the mode of this distribution?



o) The mean of a list of 10 numbers is 8. If 17 and -1 are added to the list, what is the new mean?

p) The mean of a list of 27 consecutive multiples of 5 is 425. What is the median?

q) What is $\frac{(13!)(20!)}{(26!)}+\frac{10}{23}$?

r) What is the number of centimeters in the median of the following set of measurements? Express your answer as a decimal to the nearest tenth.

$$\{126\ mm, 11.8\ cm, 1.25\ dm, 0.13\ m, 10^{-4}\ km\}$$

s) Simplify and express your answer as a common fraction: $\frac{\frac{2}{3!}+1}{\frac{1}{2!}+4}$.

t) What is the remainder when 13^{51} is divided by 5 ?

u) How many different seven-digit telephone numbers are available if the only restriction is that the first digit cannot be 0 ?

v) For how many two-digit prime numbers is the sum of its digits 8 ?

w) A magazine advertises its \$12 annual subscription price as 70% off the newsstand price. What is the number of dollars per year a reader would spend if the magazine were purchased at the newsstand price?

x) A five-digit positive integer is a "mountain number" if the first three digits are in ascending order and the last three digits are in descending order. For example, 35,761 is a mountain number, but 32,323 and 35,655 are not. How many five-digit numbers greater than 70,000 are mountain numbers?

y) A three-digit number has the same hundreds, tens, and units digit. The sum of the prime factors of the number is 47 . What is the three-digit number?

z) A pair of **emirps** consists of two prime numbers such that reversing the digits of one number gives the other. How many pairs of two-digit emirps exist such that each number in the pair is greater than 11 ?