Rules For Solving Equations: An equation is still valid if you do the following operations to it:

a) Add the same thing to both sides.
b) Subtract the same thing from both sides.
c) Multiply both sides by the same thing.
d) Divide both sides by the same thing (except 0).

Notice that (b) is actually the same as (a) (subtracting is the same as adding the negative).
Notice that (d) is actually the same as (c) (division is the same as multiplying by the reciprocal).

Ex: If $5x - 4 = 0$ what is x ?
Soln: Add 4 to both sides getting $5x = 4$.
Divide both sides by 5 to get $x = \frac{4}{5}$.

Root Of An Equation: A **root** of an equation is the same as a **solution** of the equation.
If the equation is f(x) = A(x), and if r is a root of this equation, then f(r) – A(r) = 0.

Fundamental Rule Of Equations: If $ab = 0$ then either $a = 0$ or $b = 0$ or both.
If $a^{f(x)} = 1$ then $f(x) = 0$ where *a* is any number and $f(x)$ is any combination of *x*'s.

Ex: What is x if $7^{3x+4} = 1$?
Soln: We must have $3x + 4 = 0$, so $x = -\frac{4}{3}$.

Linear Equation: A linear equation is an equation involving the first power of the unknown variable.
A linear equation has one solution. A linear equation can always be written as $ax + b = 0$.
If $ax + b = 0$ then $x = -\frac{b}{a}$.

Ex: Use the rules for solving equations to show this.
Soln: Add $-b$ to both sides (subtract b from both sides). Divide both sides by a.

Quadratic Equation: A quadratic equation is an equation involving the square of the unknown variable. A quadratic equation has two solutions (the solutions may not be different).
A quadratic equation can always be written as $ax^2 + bx + c = 0$.
Since $ax^2 + bx + c = (Ax - B)(Cx - D) = ACx^2 - (AD + BC) + BD$ one way to solve a quadratic equation is to find quantities A, B, C, D such that $AC = a$, $AD + BC = -b$, and $BD = c$.
Sometimes this can be done by inspection. If this has been done then the two solutions are $x = \frac{B}{A}$ and $x = \frac{D}{C}$, from the fundamental rule.
There is another way to solve a quadratic equation and that is to use what is known as the quadratic formula. This always works, so the quadratic formula should be **memorize**d.
If $ax^2 + bx + c = 0$ where *a, b,* and *c* are any numbers then the **two** solutions for *x* are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. If $u = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $w = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ then the original equation can be factored into $ax^2 + bx + c = a(x - u)(x - w) = 0$ which is why *u* and *w* are solutions.

Important Note: It is very easy to find the sum of the two solutions of a quadratic equation. Divide the quadratic equation through by a so that the first term is just x^2. Then the sum of the two solutions is equal to the negative of the coefficient of the x term.

Ex: What are the roots of the equation $x^2+3x+2=0$? What is the sum of the roots?

Soln: By inspection $x^2+3x+2=(x+1)(x+2)=0$.

By the fundamental rule the roots are $x=-1$ and $x=-2$. Use the quadratic formula to check. The sum of the roots equals the negative of the coefficient of x: -3 .

Ex: What is the sum of the roots of the equation $35x^2+1999x=\sqrt{423}$?

Soln: Dividing by 35 gives $x^2+\frac{1999}{35}x-\sqrt{423}=0$.

The negative of the coefficient of the x term is $-\frac{1999}{35}$. This is the sum of the roots.

Algebra 2 Homework Problems
(NO CALCULATORS)

a) A 50-lb block of cheese is cut into $1\frac{1}{4}$-lb blocks. Each small block is sold for \$4.25 . What is the total selling price, in dollars, for the 50-lb block of cheese?

b) Using 2.24 as an approximate value of $\sqrt{5}$, find the approximate value of $\frac{1}{\sqrt{5}}$ to the nearest hundredth.

c) Simplify: $(-8)^{-2/3}$.

d) Simplify: $\frac{5!\,4!}{5!+4!}$.

e) Which of the following, a, b, c, or d, does **NOT** equal the other three?

(a) $22+0.75(22)$ (b) $22(1.75)$ (c) $22(1+0.75)$ (d) $22(0.75)+1$.

f) Solve for x: $\left\{3+\left[2+\left(1+x^2\right)^2\right]^2\right\}^2=144$.

g) Express $\left(\frac{16}{25}\right)^{-0.5}+2^0+81^{0.75}$ as a mixed number.

h) What is the value of $\frac{6!}{(6-4)!\,4!}$?

i) Express as a mixed numeral: $93.\overline{3}-39.\overline{9}$.

j) Simplify: $\frac{7!\,5!\,3!\,1!}{6!\,4!\,2!\,0!}$.

k) Round to the nearest whole number: $\sqrt{72}$.

II. SIMPLIFY

l) $\dfrac{ab^{-4}}{a^{-2}b}$ m) $\left(\dfrac{x^2}{y^4}\right)^{3/2}$ n) $\left(\dfrac{s^{-1/3}t^{-2}}{u^{-4}}\right)^{-3/2}$

o) $\dfrac{(-3a)^3 \bullet 3a^{-2/3}}{(2a)^{-2} \bullet a^{1/3}}$ p) $\dfrac{(x^{-2})^{-3}(x^{-1/3})^9}{(x^{1/2})^{-3}(x^{-3/2})^5}$ q) $\dfrac{\left(x+\frac{1}{y}\right)^m\left(x-\frac{1}{y}\right)^n}{\left(y+\frac{1}{x}\right)^m\left(y-\frac{1}{x}\right)^n}$

IV. RATIONALIZE THE DENOMINATOR AND REDUCE TO SIMPLEST FORM

r) $\dfrac{1}{x-\sqrt{x^2-y^2}}-\dfrac{1}{x+\sqrt{x^2-y^2}}$ s) $\dfrac{x+\sqrt{x}}{1+\sqrt{x}+x}$ t) $\dfrac{2}{x^2-\sqrt{x^4+2x^2+1}}$

II. SOLVE FOR x:

u) $2x+3=0$ v) $\frac{5}{9}x-\frac{2}{7}=0$ w) $14x+2ab=0$

x) $x^2-3=0$ y) $(x-a)(x-b)=0$ z) $x+\frac{1}{x}=2$

aa) $x^2-5x+6=0$ ab) $x^2+x=6$ ac) $3^{x^2}=81$

ad) $27\left(3^{x^2}\right)=\dfrac{(3^x)^7}{27}$ ae) $7x^2+3x=4$ af) $17^x=1$

ag) $23^{-x^2}=1$ ah) $\dfrac{(x^2-5x+6)(x+1)}{x-2}=-3$ ai) $3=\dfrac{x-\frac{1}{x}}{1+\frac{1}{x}}$

aj) $\dfrac{1}{1-\dfrac{1}{1+\frac{1}{x}}}=-2$ ak) $|x-1|=3$ al) $|x+2|=-5$