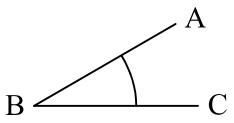
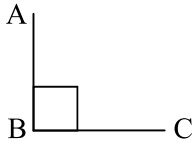
Angular Measure – Degrees: An **angle** between two intersecting lines is usually denoted with an arc as shown. **Angles** are measured in **degrees**. There are **360 degrees** (**360°**) in a complete rotation (full circle).



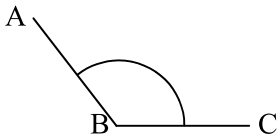
Angle: Angles are denoted in several different ways. In the figure one can refer to "angle B", or $\angle B$, or $\angle ABC$ (the middle letter is the point of the angle), or $\angle CBA$.

Right Angle: A **right angle** is a **90°** angle. Since a complete rotation is 360°, a **right angle** is ¼ of a complete rotation, as shown. A **right angle** is denoted by a square symbol, as shown.

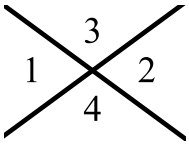


Acute Angle: An **acute angle** is an angle of **less** than **90°**, similar to the figure at the top of the page.

Obtuse Angle: An **obtuse angle** is an angle of **greater** than **90°**, as shown.



Vertical Angles: Opposite angles of two intersecting lines. Vertical angles are equal.
Ex: Angle pairs (1,2) and (3,4) are vertical angles..



Supplementary Angles: Two angles whose sum equals 180° .
Ex: Angle pairs (1,3), (1,4), (2,3), and (2,4) are supplementary angles.
Ex: If angle 1 is 98° what are the other angles?
Soln: Angle 2 = 98° since (1,2) are vertical angles.
Angle 3 = 180° – 98° = 82° since (1,3) are supplementary angles.
Angle 4 = 82° since (3,4) are vertical angles.

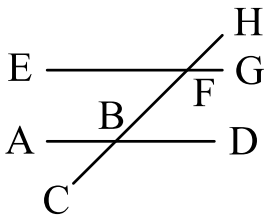
Complementary Angles: Two angles whose sum equals 90° .

Perpendicular Lines: Two lines are **perpendicular** if they intersect at **right angles**.

Parallel Lines: Two lines in a plane are **parallel** if they go in exactly the same direction. **Parallel lines never intersect**.

Skew Lines: Two lines in a plane are **skew** if they are **not parallel**.

Transversal: A **transversal** is a line which intersects two parallel lines.

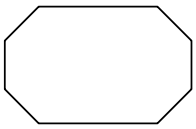


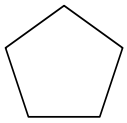
Ex: If $\overline{AD}$ is parallel to $\overline{EG}$, then $\overline{CH}$ is a transversal. If $\angle ABC = 30°$ find all the other angles.
Soln: $\angle CBD$ is a supplementary angle to $\angle ABC$, so $\angle CBD = 180° - 30° = 150°$.
$\angle ABF$ is a vertical angle to $\angle CBD$, so $\angle ABF = \angle CBD = 150°$.
$\angle DBF$ is a vertical angle to $\angle ABC$, so $\angle DBF = \angle ABC = 30°$.
$\angle EFB = \angle ABC = 30°$ (Imagine sliding $\overline{EG}$ down toward $\overline{AD}$ without rotating anything. Then no angles change and when $\overline{EG}$ meets $\overline{AD}$ the corresponding angles all match each other.) Therefore $\angle GFH = \angle EFB = 30°$, and $\angle BFG = \angle EFH = 150°$.

Polygon: A **polygon** is a figure in a plane which has N straight sides.
A corner of a polygon is called a **vertex**.
A **convex polygon** has all its sides bending toward its center at each vertex.
A **regular polygon** has all N sides the same length.

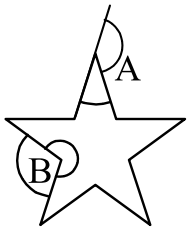

Polygon


Regular Non-convex Polygon


Convex Polygon


Regular Convex Polygon

Interior Angle: An **interior angle** of a polygon is an angle made where two sides meet measured **inside** the polygon.



Exterior Angle: An **exterior angle** of a polygon is an angle made where two sides meet measured **outside** the polygon, minus 180°.
The sum of the exterior angles of any polygon equals 360°.
Ex: In the figure shown, the interior angle A is the small angle shown while the exterior angle A is the large angle shown. The interior angle B is the large angle shown while the exterior angle B is the small angle shown, minus 180°. This exterior angle B is negative!
At any vertex of a polygon the sum of the interior and exterior angles equals 180°.

A regular polygon has N equal interior angles and N equal exterior angles.
The sum of the interior angles of an N sided polygon is $\boxed{(N-2)180°}$.
A regular N sided polygon has N exterior angles of $360°/N$. The N interior angles add to $N(180°-360°/N)=180°N-360°=(N-2)180°$. Since the exterior angles of any N sided polygon add to 360°, the interior angles of any N sided polygon must add to $(N-2)180°$.

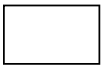
Polygon Diagonals: A **convex polygon** has $\boxed{\frac{1}{2}N(N-3)}$ **diagonals**.
Each vertex can be connected to N – 3 other vertices, and this counts each diagonal twice.

Perimeter: The **perimeter** of a polygon is the **distance along all the edges** of the polygon.

Quadrilateral: A **quadrilateral** is a **four-sided** polygon.
The sum of the four interior angles of a quadrilateral is 360°.

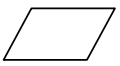
Square: A **square** is a regular quadrilateral having **90°** internal angles.

Rectangle: A **rectangle** is a quadrilateral having **90°** internal angles.

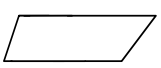


Rhombus: A **rhombus** is a regular quadrilateral.

Parallelogram: A **parallelogram** is a quadrilateral having **opposite sides parallel**.
A **parallelogram** has **opposite sides of equal length**.



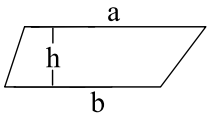
Trapezoid: A **trapezoid** is a quadrilateral having only **two sides parallel**.



Isosceles Trapezoid: A trapezoid having the **non-parallel sides equal**.

Area: The amount of **surface** space of an object.

Trapezoid Area: The **area of a trapezoid** is given by the formula (**memorize**):
$\boxed{A=\frac{1}{2}(a+b)h}$ where a, b, and h are as shown. h is the **vertical** height.



Triangle: A **triangle** is a **three-sided** polygon.
The sum of the three interior angles of a triangle is 180°.

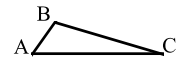
Equilateral Triangle: An **equilateral triangle** is a regular triangle.
An equilateral triangle has three equal 60° interior angles. (Why?)

Isosceles Triangle: An **isosceles triangle** has **two equal sides** and **two equal angles**.

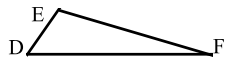
Right Triangle: A **right triangle** has **one 90° angle**. (So the other two angles add to give 90°.)
The **long side** of a **right** triangle is called the **hypotenuse**.

Similar Triangles: Two triangles are said to be **similar** if they have the **same three angles**.
Since the angles must add to 180° two triangles are **similar** if they have **two angles the same**.
If two triangles are **similar** then **their corresponding sides are proportional**.

Ex: If ∠A=∠D, ∠B=∠E, and ∠C=∠F then triangles ABC and DEF are similar.
Then AB / BC = DE / EF, AB / AC = DE / DF, and AC / BC = DF / EF.



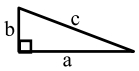
Ex: In the two similar triangles shown, if AB = 10, AC = 12, and DF = 24, what is DE ?



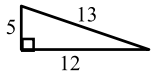
Soln: Corresponding sides are proportional: AB / AC = DE / DF so
10/12 = DE /24 so DE = 24 x 10/12 = 20.

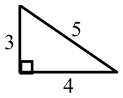
Pythagorous Theorem: In any **right** triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides. (**Memorize this!**)

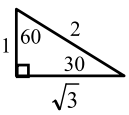
Ex: In the right triangle shown, $a^2+b^2=c^2$.

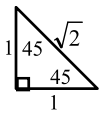


Four Special Right Triangles: (**Memorize these!**)





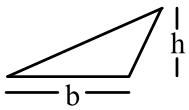




Ex: A right triangle contains a 30° angle and its shortest side is 17 feet long. How long is its hypotenuse?

Soln: The right triangle must be similar to a $(1,\sqrt{3},2)$ triangle (because of the 30° angle), so $\frac{2}{1}=\frac{x}{17}$, so the hypotenuse is 2 x 17 = 34 feet.

Triangle Area: The **area of a triangle** is given by the formula (**memorize**):



$\boxed{A = \frac{1}{2}bh}$ where the **base** b and **height** h are as shown. h is the **vertical** height. Notice that any side can be chosen to be the base b; the height h is then the vertical distance from the base to the highest point of the triangle.

Ex: What is the area of the right triangle above which contains a 30° angle?

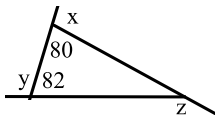
Soln: In the orientation shown, $A = \frac{1}{2}bh = \frac{1}{2} \bullet \sqrt{3} \bullet 1 = \frac{\sqrt{3}}{2}$.

Geometry 1 Homework Problems

(NO CALCULATORS)

a) If two angles are both vertical and complementary, what is each angle?

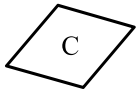
b) Find the sum of all possible values of $(x+y)$ if $x^2 + y^2 = 26$ and $xy = 5$.



c) Find the number of degrees in $x + y + z$.

d) What is the maximum number of regions in the plane created by three distinct lines, no two of which are parallel?

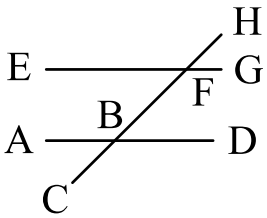
e) If the rhombus is rotated about its center point C, what is the minimum number of degrees it must rotate before it coincides with the original shape?



f) Determine the units digit of $17^{13} - 17$.

g) The smallest case of 5 consecutive odd integers whose sum is a perfect square is 1, 3, 5, 7, 9. $(1+3+5+7+9=25.)$ Find the median of the next larger set of 5 consecutive odd integers whose sum is a perfect square.

$\overline{AD}$ **is parallel to** $\overline{EG}$ **and** $\angle ABC = 47°$**.**



h) What is the value of the vertical angle to $\angle EFH$?

i) What is the value of the supplementary angle to $\angle GFH$?

j) What is the value of the complementary angle to $\angle BFE$?

II. SIMPLIFY

k) $\dfrac{\left(x^{3/4}\right)^{2/3} - \left(y^{5/4}\right)^{2/5}}{\left(x^{3/4}\right)^{1/3} + \left(y^{2/3}\right)^{3/8}}$

l) $\dfrac{1}{1+x^{p-q}} + \dfrac{1}{1+x^{q-p}}$

m) $\sqrt[x]{x^{x^2-x}}$

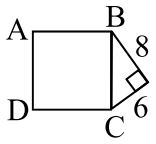
n) A triangle has sides of length 3, 4, 6. What is its perimeter?

o) A right triangle has two sides of equal length 12. What is its area?

p) What is 39×41 ?

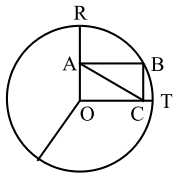
q) The diagonals of a square each have length 6. What is the area of the square?

r) The two short sides of a right triangle have lengths 20 and 37.5 . What is the area?



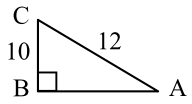
s) What is the number of square units in the area of the square ABCD ?

t) A right triangle contains a 30° angle. If the side opposite this angle is 247 feet long, how long is the hypotenuse?



u) In the figure O is the center of the circle and point B is on the circle.
Given OR = 8 and CT = 2, find the length of diagonal AC in rectangle ABCO.

v) Given isosceles right triangle ABC with leg 12. If a new, smaller similar triangle is constructed with each corresponding side one-half as long as each side of triangle ABC, the area of the new triangle is what fraction times the original?



w) Find the length of side AB. Express your answer in simplest radical form.

x) The sum of five consecutive integers is 615. What is the sum of the two smallest possible integers?

y) The product of two numbers which are consecutive multiples of 5 is 500. What is the smallest possible of the two numbers?

z) If a rectangle is formed by doubling the length of one side of a square and halving the other side of the square, what is the ratio of the perimeter of the rectangle to the perimeter of the square?