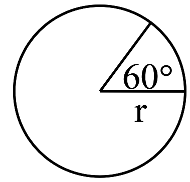
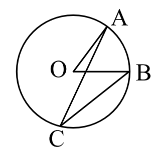
Subtended Angles: An angle **subtended** by two lines is the same as the measure of the angle of intersection of the two lines.



Arc Length: The **length of arc** subtended by two radii of a circle equals the fractional amount of circumference subtended by the two radii.

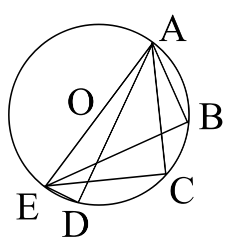
Ex: What is the length of arc of a circle of radius 36 cm subtended by two radii making an angle of 60° with each other?

Soln: The circumference is given by $2\pi r$. This is over a 360° angle. The fractional part subtended by the two radii is $\frac{60}{360} \bullet 2\pi r = \pi r / 3 = \pi \bullet 36/3 = 12\pi$ cm.



Subtended Angles In Circles: If two straight lines meet on a circle and subtend a certain arc along the circle, then the angle of intersection of the two lines equals one-half the angle subtended by two radii which subtend the same arc.

Ex: In the figure lines AC and BC meet on the circle at C. They subtend the arc AB along the circle. Lines OA and OB are radii subtending the same arc AB. Then $\angle ACB = \frac{1}{2} \angle AOB$



Any two lines drawn from the ends of a diameter of a circle which meet on the circle intersect each other in a right angle.

The ends of a diameter subtend an angle of 180° on the circle. Therefore, lines from the ends of the diameter which meet on the circle must subtend an angle of $\frac{1}{2}(180°) = 90°$.

Ex: In the figure AE is a diameter of circle O. Then $\angle ABE = \angle ACE = \angle ADE = 90°$.

Congruence: Two objects are said to be **congruent** if they can be rotated or flipped so that they match up perfectly with each other.

Similarity: Two objects are said to be **similar** if they can be uniformly stretched so that they are congruent.

Proportion: Using **proportions** can often make the math simpler and easier.

Direct Proportion: $\boxed{a = Kb}$ where a and b are variables and K is a constant for this problem.

If a is **directly proportional** to b then $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2}}$.

Inverse Proportion: $\boxed{a = K / b}$ where a and b are variables and K is a constant for this problem.

If a is **inversely proportional** to b then $\boxed{\frac{a_1}{a_2} = \frac{b_2}{b_1}}$.

Ex: The stretch in a wire is directly proportional to the weight on the wire. A weight of 10 lb produces a stretch of 2 inches. How much stretch does a weight of 35 lb produce?

Soln: $\frac{s_1}{s_2} = \frac{w_1}{w_2}$ so $s_1 = s_2\left(\frac{w_1}{w_2}\right) = 2\left(\frac{35}{10}\right)$ = 7 inches.

Areas Are Directly Proportional To $(\text{Length})^2$.

Volumes Are Directly Proportional To $(\text{Length})^3$.

These obvious statements can produce very powerful results.

Ex: A square pyramid has a base edge of 32 inches and an altitude of one foot. A square pyramid whose altitude is one-fourth of the original altitude is cut away at the vertex. The volume of the remaining frustum is what fractional part of the volume of the original pyramid? (This was a real problem!)

Soln: I may not know a formula for pyramid volumes. However, I do know that $V = KL^3$, where K is some combination of angles and length ratios depending on the particular solid in the problem. Since a pyramid has everything linearly related (all edges are straight lines) the little pyramid is just a small copy of the big pyramid. Therefore, the K for the little pyramid is the same as the K for the big pyramid. Let V_1 be the little pyramid volume and V_2 be the big pyramid volume. Let L_1 be the little pyramid altitude and let L_2 be the big pyramid altitude.

Then $\dfrac{V_1}{V_2} = \left(\dfrac{L_1}{L_2}\right)^3 = \left(\dfrac{1}{4}\right)^3 = \dfrac{1}{64}$.

Therefore, the remaining volume fraction of the original pyramid is $\dfrac{63}{64}$.

Scaling: One object is said to be **scaled** from another object if the two objects are **similar**.

Units: When **multiplying** quantities having **units**, the units of the product equals the product of the units.

When **adding** quantities having **units**, each term in the sum must have the **same units**.

Using Data To Find Constants In A Relation. If you are given an equation containing constants that some data is supposed to satisfy, then you can use the data to evaluate the constants in the equation. You need to use as many different bits of data as there are constants in the equation.

Ex: The table shown is a linear relation between x and y of the form y = ax + b. What is the value of b?

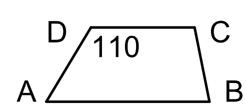
x	1	2	3	4	5
y	10	17	24	31	38

Soln: The equation contains two constants a and b so you need to use two different bits of data. Choose simple data numbers (x,y) = (1,10) and (2,17). Then 10 = a + b and 17 = 2a + b. Eliminate b by subtracting the first equation from the second equation to get a = 7. Then b = 10 – a = 3 .

Special Numerical Values: (memorize) $\boxed{\sqrt{2} \cong 1.41}$ $\boxed{\sqrt{3} \cong 1.73}$

Geometry 3 Homework Problems
(NO CALCULATORS)

a) What is the number of degrees in the measure of $\angle A$ in the trapezoid?

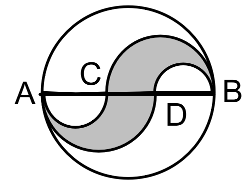


b) How many positive integers have cube roots less than $1+\sqrt{2}$?

c) The force of gravitational attraction between two objects is inversely proportional to the square of their distance apart. If two objects have a gravitational force of 750 units when they are 3300 meters apart, how far apart are they when their gravitational force is 1000 units? Express your answer to the nearest meter. (You may use a CALCULATOR for **this** problem.)

d) A wrecker's iron ball eight inches in diameter weighs 120 pounds. How many pounds would a similar iron wrecking ball twelve inches in diameter weigh?

e) The diameter $\overline{AB}$ of the circle in the diagram is trisected such that $AC = CD = DB$. Semicircles are determined by $\overline{AC}, \overline{AD}, \overline{CB}, \overline{DB}$. What fractional portion of the large circle is the shaded area?



f) Point Z is the midpoint of $\overline{MN}$. Point P is the midpoint of $\overline{ZN}$. Point Q is the midpoint of $\overline{PN}$ and point R is the midpoint of $\overline{QN}$. If $\overline{PR}$ is 24 inches long, how many feet long is $\overline{MN}$? Express your answer as a mixed number.

g) The radius of a right circular cylinder is decreased by 20% and its height is increased by 25%. What is the absolute value of the percent change in the volume of the cylinder?

h) Find the geometric mean of $6\frac{1}{4}$ and 100.

i) The five tallest buildings in Los Angeles in 1985 had a mean height of 733 feet. The tallest of the five buildings had a height of 837 feet and the shortest of the five buildings had a height of 715 feet. If a new building were constructed with a height of 855 feet, by how many feet would it increase the mean height of the five tallest buildings in the city?

j) A piece of wood is carved in the shape of a pyramid with a pentagonal base. What is the sum of the number of faces, the number of edges, and the number of corners of the pyramid?

k) A pyramid has a base shaped like a regular hexagon. Each side of the hexagon has length 10 cm. Each sloping edge of the pyramid has length 20 cm. What is the volume of the pyramid?

l) A circular hoop has a circumference of 6π cm. What is the volume of the largest ball that will just pass through the hoop?

m) Two trains are at the same station. Train A goes due East at 20 miles per hour. One hour later train B goes due North at 30 miles per hour. How far apart are the trains when train B has been traveling for one hour?

n) The table represents the relationship between x and y in a quadratic equation of the form $y = ax^2 + bx + c$, where a, b, and c are integers. What is the value of a?

x	1	2	3	4	5	6	7
y	-8	5	24	49	80	117	160

o) Evaluate and express as a common fraction: $(16)^{-1/4} - (8)^{-2/3}$.

p) The measures of a pair of supplementary angles are in the ratio of 7:2. How many degrees are in the measure of their positive difference?

q) Divide and express your answer in scientific notation: $\frac{2\times10^{13}}{4\times10^{10}}$.

r) The sum of the digits of a two-digit number is 12. When the digits are reversed, the original number is increased by 54. What is the product of the digits?

s) There exist non-negative integers a and b such that for any x satisfying $a < x < b$, it is true that $\sqrt{x} > x$. Find the sum of the ordered pair (a,b).

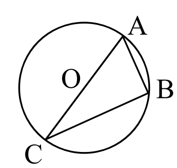
t) A group of boys and girls reserved a single row of 120 seats at a theater. The girls were assigned seat numbers in such a way that every boy had to sit next to at least one girl. What is the fewest number of girls necessary to make this happen?

u) Find the value of K for which these two equations will not have a common solution: $6x + 4y = 7$ and $Kx + 8y = 7$.

v) Two factors have a product of $3 \bullet 9^{2x} - 6 \bullet 27^{x}$. One of the factors is $3^{x} - 2$. What is the other factor?

w) Solve for n: $\frac{8!}{n!} = 336$.

x) Nolan Ryan holds the record for the fastest pitch in baseball. He pitched a ball at a speed of 100.9 miles per hour. How many seconds, to the nearest tenth, did it take Ryan's pitch to travel the 60 feet, 6 inches from the pitcher's mound to home plate? (You may use a calculator.)



y) If AOC is a diameter of circle O of radius 15 cm, what is the area of triangle ABC if side AB is 9 cm long?

z) In the figure shown let the diameter of circle O be 16 cm. What is the arc length AB if angle ACB is 35° ? Express your answer as a fraction in terms of π.